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DIO

Archimedes as Astronomer

$2200^{\rm y}$ Old Sunsize-Disguise — Fractions Mask Degrees

Hipparchos

Faked 179° True-Elongation $_{-}$381/12/12 Lunar Eclipse

His 2 Indoor $_{-}$157 Solstices

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"News": One-Syllable Word for "Propaganda". Three PerfectgameStreaks. TV 'snews is setting new records in shamelessness, adhering to propagandists' 1st Commandment: don't over-lie, but selectively censor truths upsetting to moguls. (Like establishment goons who honor *DIO*'s reliability&impact by religiously serial-deleting from Wikipedia our otherwise-unanswerable exposés of archons: www.dioi.org/dec.htm#wdpq.)

[A] **Don't do Neighborhood Watch in an electionyear. Energize-the-base, anyone?** The latest is a first, a network's persistent agitprop-fabrication of a mythical event: the Sanford FL "murder" by punched-bloody neighborhood-watcher George Zimmerman,[1] of tall black bully Trayvon Martin, canonized by Ed Schultz of NBC (the Obama network)[2] as our era's symbol of the civil-rights movement! NBC punditz dissed the very idea GZ had wounds; Dembo Larry O'Donnell stooped to GrassyKnollesque paranoia on videos of GZ's head, diverting from THE uncited crucial-test datum: besides the fatal bullet, ***there are no serious wounds on Martin***. Plainly the case's resolution. Except: no one mentioned it. Week after week. How DO the nets do it? Like a pitcher tossing a perfectgame shutout everyday for months. . . . Dave Barry asks the purpose of gov't. His tart answer: to Tax. *DIO* asks [www.dioi.org/pre.htm]: What is the purpose of US newsmedia? Answer: BIAS.

[B] **"Operation Iranian Freedom"? Santayanan WMD&OilCartel-Altruism Replay?** The most conspicuous aspect of the war-cry to raid Iran is the unconspicuousness of media-alerts on the plain parallel to the 2003 pseudo-preemptive raid of Iraq on equally shaky & convenient intel on supposed WMDs. (Maybe much-fed via Iran's secret service, to sucker Divine-Flounder legacy-prez Shrubya into offing Iran's top local enemy.) If Iran is building a military-nuke at all, it's not for starting a suicidal war but rather as a mutual-blackmail defense against getting attacked, the very tactic the US&Russia have carried on for years. Why is an ever-more-oil-addicted world trying to add Iran-grab to Iraq-grab? Because Iraq & Iran are the world's biggest oil puddles (besides stoner-age ally Saudi Arabia)? No, it's Nationbuilding. (Which if it westernizes $7\mathrm{x}10^9$ souls' lifestyles, quickburns all Earth's oil.)

[C] **Another Perfect-Game Skein. Marlowe→Shakespeare Shutout in World Press.** The zany film *Anonymous* got 1 thing right: actor-loanshark Shakespeare didn't write plays. The 2011 October spate of pans of the film raised a parallel joke-question: did one hand write them all?! Were **all** reviewers for the usual establishment-servile forums really as innocent of the truth as the public they so crucially&leaklessly keep in like darkness? The evidence that Christopher Marlowe wrote Shakespeare's plays is so simple as to be reduceable to a single paragraph (*DIO 18* §*A3*). Will's front is tied with F.Cook's N.Pole-prank as the most transparent hoax DR ever looked into. Which is why the Stratfordian myth cannot survive unless the wider public is NEVER informed of said evidence. Luckily, our echohead-press' Oct shutout shows it's fully&foully up-to its Horatius@Bridge rôle.

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[1] Even after bail-hearing disaster, NBC national news (4/23) still painted GZ as liar. Uncited clues to the truth: Which party pre-called the cops? Which had bloody beating-marks? Fearing cred-damage, the nets are presumably influencing GZ (Fuhrmanesque book-deal?) to help equi-blur the unequal truth. (Whyelse suddenly switch to a lawyer related to CNN, one of the nets lynching him for weeks? Will Martin's autopsy be hid as long as Skip Gates' full police-tape?) GZ rightly profile-spotted one whose reaction to neighborhoodwatch's chief was: aiding the robust local burglar-guild by assaulting him. (Has any newsman cared aloud about the potential future beating-victims spared by this "tragedy"?)

[2] Racecard-pros got a national-TV freehand for weeks, *even while* anyone pro-GZ was accused of the sin of trying-the-case-in-public! It doesn't get more perverse. Or more insulting to black IQ. Lies that GZ "hunted" TM as a "coon" were repeatedly broadcast as-fact. MSNBC&CNN promoted confused testimony and bozo "Voice Experts", even doctoring a GZ-tape, all to inflame mass racial paranoia (to Dembos' November profit) by airing black-racist dream-come-true "proof" of The Ultimate Evil: *racial profiling*. (DR has had his papers checked in Europe twice and seen many other such profilings. It helps maintain the good life there, at rare&minor inconvenience compared to that caused by massive criminality.) But to TV 'snews, bruised feelings from profiling are of greater and far more frequent hysterical concern than racial inequities in trifles like theft, drugs, poverty, bastardy, illiteracy, murder.

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Most unattributed text is DR's.

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Free spirits will presumably be pleased (and certain archons will not be surprised) to learn that: at *DIO*, there is not the slightest fixed standard for writing style.

Contributors should send (expendable photocopies of) papers to one of the following *DIO* referees — and then inquire of him by phone in 40 days:

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of finding no *DIO* math error, alternate eclipses, or equally accurate methods that *could match DIO by eliciting all eight of Hipparchos' digits*. Exactly.[27] Again: née-jerk archons yet lack the sense&balance to gauge, face, or admit the obvious uniqueness of our solution to one of the key equations in all ancient astronomy. In other words: the usual.

L5 We remember also outdoor-Hipparchos' establishment of the earliest extant grand star catalog, a treasure stolen by indoor-Ptolemy, but lately restored to its rightful creator, through researches (R.Newton 1977, Rawlins 1982C, Graßhoff 1990, Rawlins 1994L, Pickering 2002A, Duke 2002C) none of which appeared in hist.astron's centrist & "premier" journals, who instead spat on the truth for decades (1975-2002), to the extent of scores of pages of reliably humorous pseudo-scholarship. (Highly admirable Muffia exception: Toomer's swift conversion by and publication of epochal Graßhoff 1990 via Springer.)

L6 The present article and the previous are our latest installments in *DIO*'s ongoing (from *DIO 1.1* to date) expanded view of Hipparchos' evolution from amateur (c.−160) into a serious contributor (c.−130) to the growth of astronomy. Our journey has been, like his and his science's, a Niagara of surprises: the irresistible lure of the inductive journey.

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[27] This holds for all three of *DIO*'s solutions of the origins of anciently-adopted lunar speed estimates: ***twenty-four digits*** in all, each reproduced exactly. Details: www.dioi.org/thr.htm & *DIO 16* p.2.

This ***DIO*** is dedicated to the ever-treasured memory of Charlie Kowal (1940-2011) — irreproducible genius, celestial&historical discoverer, brave&principled friend.

‡1 Archimedes' Hidden Measure in Degrees — Sunsize Disguise: His Solar Diameter

Hellenistic Astronomers' High-Empiricism Confirmed by Accurate Solar Brackets

Babylonian Degree-Measure Already Greek-Adopted by 3rd Century BC

A Summary

DIO has recently discovered that Archimedes' *Sandreckoner* estimate of the Sun's diameter — which has the surface look of a crude, pedant-conventional unit-fraction range — was in truth a professional-level empirical measure, expressed instead in the sexagesimal convention of 3^{rd} century BC astronomers: $30' \pm 3'$. Accurate. And couched within reasonably-cautious — and correct — uncertainty-limits. Ultimate resolution below at §E.

B Did 3^{rd} Century BC Greek Scientists Use Degrees? Dating Strabo's Nile Map

B1 In recent years, the superficially ambiguous evidence regarding when the Hellenistic astronomical tradition adopted Babylon's sexagesimal measurement of angles in degrees, arcmin, etc, has led several able, prominent scholars (including sometime-Princetitutees) to doubt or cite doubt that 3^{rd} century BC Greek astronomers used degrees (Dicks 1966 n.15; Jones 1991M n.5; B.Goldstein & Bowen 1991 pp.103-105; van Brummelen 2009 p.33 n.2).

B2 By contrast, *DIO* has repeatedly pointed out (e.g., Rawlins 1991W fn 53, Rawlins 1994L fn 41, & Rawlins 2008R fn 24) the probability of degrees' use by said astronomers.

B3 But, ironically, DR's own Rawlins 1982N paper perhaps added to the confusion since it showed that the Eratosthenes Nile Map relayed by Strabo 17.1.2 used (instead of degrees) successive halvings[1] of circle-fractions. The exact date of the map is not known; however, the map obviously (Rawlins 2008Q eq.11) post-dates the Alexandria Lighthouse — and pre-dates the end of Eratosthenes' career. Which sandwiches the map into the period c.270-200 BC, dating it for the 1^{st} time. The Nile Map's unit was a Pharos-based (Rawlins 2008Q) Earth-radius probably due to the Alexandria Lighthouse's architect, Sostratos:

$$\text{Sostratos Earth circumference } C_{\text{S}} = 256000 \text{ stades} \tag{1}$$

which is 19% too high — but for Pharos-based Earth-measure, *we expect 20% excess due* to air's bending of horizontal light-rays (to a curvature equal to 1/6 of that of the Earth's surface), so the tight match evidences high precision (half-percent: *ibid* §I3) empirical measurement by Greek scientists (*ibid* §K4), Details at *op cit*.

[1] The Nile Map is based upon successive halvings of $7^\circ 1/2$. Note parallel at ‡3 fn 16.

B4 Eq.1's C_{S} was ultimately adopted by Eratosthenes but manipulated to (Strabo 2.5.7):

$$\text{Eratosthenes Earth circumference } C_{\mathrm{E}} = 252000 \text{ stades} \tag{2}$$

— the canonical "Eratosthenes" circumference, his slight alteration (less than 2%) perhaps effected to arrange an even multiple of 360, so that for Eratosthenes

$$1^\circ = 700 \text{ stades} \tag{3}$$

suggestive[2] of Greek scientists' adoption of the degree at least by the mid-3rd century BC.

C Earlier Evidence

C1 When we began looking for evidence bearing on the era when Greeks started precisely measuring celestial coordinates in degrees, the earliest datable clue encountered was Ptolemy's collection of 18 stellar declinations — 12 by Timocharis & 6 by Aristyllos — observed in Alexandria c.300 BC & c.260 BC, resp (Rawlins 1994L Table 3) and all expressed in degrees at *Almajest* 7.3. Particularly striking is the uniform rounding by Aristyllos of all six of his star declinations to $1^\circ/4$ precision: a conscientiously cautious practice — perhaps intended to avoid erroneously reporting slightly discrepant empirical results, but which may (Standish 1997 DR Comm. §G12) have cost him discovery of precession, an honor which (Rawlins 1999 §D5) should instead go to his contemporary, Aristarchos. The obvious question regarding Aristyllos: if his declinations were originally reported in some other measure than degrees, how likely is it that, after hypothetical subsequent transformation (into the degree-values reported by Ptolemy at *Almajest* 7.3), all six data would[3] end up exhibiting consistency with quarter-degree rounding?

[2] Early Greeks divided Earth-circumference C into 60 parts (Strabo 2.5.7 & Neugebauer 1975 p.590 n.2), not 360, nor the public-treatise custom of quadrant-unit-fractions (Aristarchos&Archimedes: *idem* or our eq.6). [Note added 2014/4/29&2017/7/1.] So ***was the stade defined sexagesimally***? Like our nautical mile (C/21600)? Or the meter (C/[4x10^7])? Greeks expressed fractions sexagesimally. Was the formerly-unsteady, locally-varying stade imperially regularized c.300 BC as 1/60th of 1/60th of 1/60th of 1^{C}? Above-***attested*** $1^{\mathrm{C}}/60$: cascade's *determinant step 1*. Our proposed ***integral unit-fraction*** in modern sexagesimal notation: 1 stade ≡ PRECISELY 0^{C};00,00,01 (thus $1^\circ \equiv 600$ stades). At Egypt, latitudinal Earth-curvature's $C \doteq$ 39,900 km: $C/60^3 = 184.7$ m $\doteq$ standard Alexandrian 185 m stade, independently-validated at Rawlins 2008Q §K2. Pharos shows the early Ptolemaic empire's enthusiasm for vast projects. Did an ordmag-1000-mile version of the unsubtle but low-systematic-error Kleo-Method (Rawlins 2008Q §A4; *Geogr.Dir.* 1.3.2-3), and/or a royal surveyors' project, find the equivalent of correct $C = 39900000$ m, thus 1 stade $\equiv C/216000 = 185$ m, well before Sostratos and Eratosthenes got clever with measurement by the Pharos' flame? See www.dioi.org/cot.htm#csqm.

[3] We can test independently whether Aristyllos (c.260 BC) used 60ths (fn 2) instead of 360ths of a circle for his share of the only 18 Greek declinations surviving from the 3rd century BC: those 6 in the north quarter of the sky. Try his ζ UMa declination, where *Almajest* 7.3 reports $\delta = 67^\circ 1/4$, which — in circle-60ths — exactly translates to $11^{\mathrm{x}}5/24$ (not credible) or $11^{\mathrm{x}}12'1/2$ (too precise, compared to the simplicity of $67^\circ 1/4$). Testing instead $11^{\mathrm{x}}12'$ & $11^{\mathrm{x}}13'$, we find that they would result in $67^\circ 1/5$ (or 1/6) & $67^\circ 1/3$, respectively — not $67^\circ 1/4$. So this star alone eliminates the theory that Aristyllos used degree-60ths. B.Goldstein & Bowen 1991 pp.103-105 suggest he could have used two-degree "cubits" or degrees — or half-degree "points" or half-degree "Moon-breadths". But the differences between these measures are too trivial to regard as generic. Also: [a] two-degree cubits (*ibid* p.104) would require overprecise 13 1/2 & 16 1/2 for Archimedes' solar brackets; and even more unlikely: 33 5/8 for Aristyllos' ζ UMa declination. [b] Ancient mention of points is later than of degrees. [c] As for Moon-breadths ($1^\circ/2$): these were just visual yard-sticks for eyeball-observers who lacked the ringed astrolabe — irrelevant to Aristyllos' transit-instrument observations. Further: Moon-breadths are also Sun-breadths. Wouldn't it seem a mite superfluous for Archimedes to announce that he had empirically measured the Sun's width to be one Sun-width? (And: how do you call a quantity equal to itself plus-or-minus 10%?)

L2 But (as noted at *idem*) there is a striking and **perfect** correlation here (which reminds us that ephemerides FOR a given year are frequently computed AT a quite different year): three of the six texts do indeed use eq.18 ***but they are not dated on the clay*** — while all three of the texts which ARE dated on-the-clay (dated as created c.200 BC) do NOT use eq.18. Ah, but there is a 7th text which is clay-dated — and uses eq.18. So that clinches it for the centrists? No — ITS date is well after Hipparchos. (Full discussion: Rawlins 2002H §D.) So the supposed impediment (to accepting *Almajest* 4.2&9's attribution of eq.18 to Hipparchos) actually just adds — 7-times-out-of-7 — to the pro-Hipparchos evidences. These evidences were already manifold, precise, and in some cases jaw-dropping.
E.g., eq.18 can only be based on the standard Greek method of finding lunar motion (namely, eclipse-cycles) if an ***apogee-perigee*** eclipse-pair was used (Rawlins 2002H §B):

$$13645^{\mathrm{u}} = 14807^{\mathrm{w}}1/2 = 14623^{\mathrm{v}}1/2 + 7^\circ \tag{19}$$

L3 Division by 5/2 (like §I5) produced eq.18. Clincher: the only eclipse-analyst known to make the highly peculiar choice of apogee-perigee pair is Hipparchos. And he is attested (*Almajest* 6.9) as doing so using the very −140/1/27 perigee lunar eclipse which Rawlins 2002H showed would ***precisely account for all eight digits*** in eq.18 if Hipparchos compared it to a (since-lost) record of −1244/11/13's apogee eclipse (instead of comparing it to the −719/3/8-9 apogee eclipse he used for his 1st try: *Almajest* 6.9). We know period-relation eq.18 is not from predecessors (as at §J) since he saw the −140 eclipse (*Almajest* 6.5&9). Rawlins 2002H §C provides a detailed survey of the ***SIX-FOLD*** array[23] of such testimonial, methodological, & quantitative verifications of Hipparchos' authorship of eq.18.

L4 Discovery of precession is commonly mis-attributed to Hipparchos, though it was undeniably (Rawlins 1999) known to Aristarchos over a century earlier. So eq.18 is easily Hipparchos' greatest scientific discovery. Rather than subtle math, finding eq.18 primarily required dedicated determination: laboriously filtering extremely ancient records (over 1000^{y} old to him — obviously part of "the series[24] brought over from Babylon": *Almajest* 4.11) added to Aristarchos-level (www.dioi.org/cot.htm#tqdr) fine judgement in eclipse-choice. The result was (& *is*) accurate to 1 part in ordmag *ten million*. And not by accident. So, when considering Hipparchos' lesser moments (due to the math limitations of himself & his colleagues, especially early on), we should keep in mind his marvelous eq.19→eq.18 advance of lunar theory,[25] a decade-old *DIO* discovery (Rawlins 2002H) still nowhere even understood (much less accepted) by hist.astron archondum, which gave up fighting us after instant-torpedo-reversal (§L2) produced only the dreary longterm downer[26]

[23] Impervious to every item of this devastating series of childishly obvious indicia, our ever-ineducable *DIO*-denier klan instantly hurled its six-texts killer-wannabe torpedo (§L1) at *DIO*. When that shot just-as-instantly backfired (§L2), *DIO* was hugely & at-the-time-gratefully (Rawlins 2002H §D1) enlightened. But, again: Muffiosi learned nothing — and just skulkingly departed discussion *without a word on their latest ideakiller-dud*. It's so efficient&comforting — and so like C.Ptolemy — to know answers ahead of incoming evidences, regardless of blow after blow after blow after blow of such.

[24] The early eclipse records from Babylon may have been pretty dense at least in patches. How else explain that Hipparchos was able to search through and find a just-right match to establish eq.18?

[25] J.C.Adams' 1846 Neptune fiasco (Rawlins 1999N) was followed by redemption through his brilliant, ultimately fruitful pioneering discovery (Rawlins 1992W §I12) of lunar theory's discord with observation, due to Earth-spin acceleration, the prime research field of the eminent Johns Hopkins physicist R.Newton. This work eventually led him to expose Ptolemy's fraudulence (e.g., R.Newton 1977), triggering decades of ethically repulsive AmerAstronSoc-HAD-*JHA*-Muffia denigrations, non-citations, suppressions, & shunning of him, an academic obscenity now surviving by transference of target to DR. None of which appears to upset any of those vaunted watchdogs we keep hearing about, whenever archons try to convince Congress that academic misbehavior is of trifling dimensions.

[26] But the frustration-downer for seething semi-numerate archons is matched by a kinda-upper for *DIO*. We prefer & repeatedly invite communication and-or (www.dioi.org/deb.htm) debate even with the doltiest of the hist.astron field's archons. But, failing that, it is tragicomically entertaining to watch their political obsessions retard their own field's progress by barring valid scholarship — thereby (as we predicted two decades ago: Rawlins 1991W §P3) "betraying their very profession. Not every scholar's detractors are so obligingly cooperative in thus destroying their own intrinsic credibility."

adducement of 3162 (yet-another goal-directed special assumption) is justified by some way-later (obviously not precision-addicted) Indian astrologers' use of $\sqrt{10} = 3.162$ for π. (Unheard-of for Greek astronomy; & *extant Indian records nowhere exhibit* $R' = 3162$.)

K6 In the step that produces r (§I12 step 3), instead of following Toomer's arbitrarily freezing the numerator (231.727, after §I15's correction) it's instead *differently*-arbitrarily treated: merged with its denominator (c.2960) and the result multiplied by $d = 3162$:

$$r = 1000 \cdot \sqrt{10} \cdot 231.727/2960.39 = 247.53 \doteq 247\ 1/2 \qquad (15)$$

The equation for R (Toomer 1984 p.11) then nearly becomes (Duke 2005T p.173):

$$R = \sqrt{3162^2 + [247\ 1/2]^2 - 3162 \cdot 106\ 1/7} \doteq 3122\ 1/2 \qquad (16)$$

But eq.16 doesn't work as printed (at *ibid* p.173): 106 1/7 is a harmless remnant resulting from one of a succession of shopping-trips[22] seeking a way of fitting the calculation to Hipparchos' results. (Back when $d = 3438$ was tried-out before going for 3162.) Once we switch fully over to $d = 3162$, the intended equation (which Duke actually&correctly computed with) appears [though exact math all the way yields c.3122 2/3]:

$$R = \sqrt{3162^2 + [247\ 1/2]^2 - 3162 \cdot 97\ 2/3} \doteq 3122\ 1/2 \qquad (17)$$

L Hipparchos in Toto

L1 *DIO* has defied 100^{y} of Experts' mis-ascribing to Babylon Hipparchos' amazingly accurate draconitic period-relation:

$$5458^{\mathrm{u}} = 5923^{\mathrm{w}} \qquad [= 5849^{\mathrm{v}} + 147^\circ] \qquad (18)$$

(Where we've appended in brackets the seemingly hopeless anomalistic situation, for contrast with near half-integrality below at eq.19's ***precise*** resolution of the mystery of eq.18's origin. Superscripts: u = synodic months, v = anomalistic months, w = draconitic months.) Said consensus asserts that Babylonians used this equation c.200BC (long before Hipparchos), citing 6 cuneiform-text lunar data (Rawlins 2002H §D1) calculated for that time.

$R' = 3438$ is found by dividing a 4-place-accurate value for π into 10800.

[22] Another symptom of the flexibility of the search for a path to a fit: *Centaurus* also missed two places where a choice for chord-table-radius R' was in-flux — thus left for later filling-in, but never got filled. At p.164, we read that Toomer's "hypothesis that Hipparchus had used a chord table of radius was correct". And at p.172: "following the same path as in Trio A using does not give". The omissions are again harmless, but together they show that not only did *Centaurus* check none of the math, it didn't even read the text. Worst: *Centaurus* didn't require citation of Rawlins 1991W, and-or its prominent summation by Thurston 2002S (nor did *JHA* for the Babylonianist-Muffiose paper Duke 2008W pp.290, 293, 294 & nn.22-23; on n.24, see *DIO 6* ‡3 §§D&H2!!), which had ***historically*** (*ibid* §D6) recovered all 4 Hipparchos numbers exactly; this, even though Duke 2005T **was born to bury Rawlins 1991W**. (Duke 2005T started a decade ago as an entry for *DIO*'s van der Waerden Award, which invites [www.dioi.org/pri.htm] such challenges.) But, without Duke 2005T's stimulus (plus Toomer 1973's & Jones 1991H's, earlier), DR probably wouldn't have ever discovered this paper's most startling new result (§G1), neatly consistent with the §E7 Pair Method hypothesis: that Hipparchos' school had — due to problems with said method — clumsily (Rawlins 1991W §N15) faked a 179° true-elongation lunar mid-eclipse. That the legendary "Father of Astronomy" was involved in fraud merely adds to the list of eminent figures whom *DIO* has investigated in connexion with such activity. E.g., Ptolemy (Pickering 2002A, Duke 2002C), Tycho Brahe (Rawlins 1992T), Marlowe-Shakespeare (www.dioi.org/sha.htm), A.Robertson (Standish 1997), Isaac Hayes (www.dioi.org/hay.htm), R.Peary (Christiansen 1997), R.Byrd (www.dioi.org/byf.htm). [Also: fake-refereeing at some extremely handsome journals, e.g., *Centaurus* (just above; or fn 17) and Lord Hoskin's shunloving (Rawlins 1991W §B3) *Journal for the History of Astronomy* (www.dioi.org/jha.htm#kqlz).] But most of these figures left more positive than negative legacies. Is there a correlation between [a] exaggerations too often attendant to fundraising needed for great deeds, & [b] sham that also-too-often attends moments of falling short of greatness?

C2 The next point coming to our attention (noted at Rawlins 2008R fn 24) was Aristarchos' record: his 1/720 of a circle for solar-diameter (eq.7; Rawlins 2008R eq.3), which is 1°/2; his empirical estimate (his Hypothesis 4) that half-Moon elongation was 1/30 of a RtAng from quadrature (*ibid* eq.4 or Heath 1913 pp.352-353), which is 3°; and Ptolemy's mention (*Almajest* 4.2) that in the Aristarchos luni-solar scheme (Rawlins 2002A), the saros' excess over 18^{y} was 10°2/3 — that is, 32° excess for the 54^{y} exeligmos — an integrality which led to *DIO*'s reconstruction (*idem*) of the origin of Aristarchos' monthlength:

$$M_{\mathrm{A}} = 29^{\mathrm{d}}31'50''08'''20'''' = 765433^{\mathrm{d}}/25920 = 765433^{\mathrm{h}}/1080 \qquad (4)$$

(falsely labelled by hist.astron's political-centrists the "Babylonian" month though unattested in Babylon before c.200 BC), based upon the 4267-month eclipse-cycle (as correctly reported by Ptolemy at *Almajest* 4.2), a value accurate to a fraction of a timesec then and now. (For other viewpoints, see, e.g., Swerdlow 1980 & Engelson 2006A.) That's one part in several million, and it's based upon degrees not only in the expressions for saros & exeligmos but — as pointed out to DR by John Britton and John Steele — in the key rounding (Rawlins 2008Q §A8) that produces the precise degree-expression of Aristarchos' monthlength in its original form:

$$M_{\mathrm{A}} = 29^{\mathrm{d}}191^\circ 00'50'' \qquad (5)$$

which later became equivalently expressed in the sexagesimal format (eq.4) we know from *Almajest* 4.2.

D Archimedes the Astronomer

D1 Archimedes is not usually seen as astronomer but as combo of mathematician and arms-designer. Yet the latter career could hardly have occurred without a scientist's knowledge, drive, and thought-habits.

D2 *Almajest* 3.1 quotes Hipparchos' testimony that he & Archimedes observed solstices to an accuracy no worse than $1^{\mathrm{d}}/4$, so we know that Archimedes had 1$^{\mathrm{st}}$hand outdoor experience in solar work.

D3 His measure of the Sun's diameter (to be analysed in what follows) gives flesh to that supposition, as well as providing a prime example of ancient scientific writers using circle-fractions for publication of empirical data actually measured in degrees, a more familiar example of which is Eratosthenes' description of the Earth's obliquity as 11/83 of a semi-circle, when (*Almajest* 1.12) $23^\circ 51' 1/4 \pm 1' 1/4$ was the actual (precise but inaccurate: Rawlins 1982G eq.9) mean measurement by asymmetric (unfortunately) gnomon.

D4 Archimedes' report (Archimedes p.224) is that the Sun's angular diameter $d_\odot$ is between 1/200$^{\mathrm{th}}$ and 1/164$^{\mathrm{th}}$ of a quadrant — a right angle or 90°. Which can be expressed thusly:

$$\text{Archimedes:} \quad \mathrm{RtAng}/200 < d_\odot < \mathrm{RtAng}/164 \qquad (6)$$

What can a description of such oddity (§E1) be telling[4] us?

[4] However, one must be taught BY evidence rather than teaching TO it. E.g., Shapiro 1975 p.77 long ago realized that exact conversion of Archimedes' brackets equalled 27′ & 32′56″. But he then spurned the discovery-opportunity here by neglecting to ponder: [a] the latter angle's glaring nearness to 33′, [b] the pair-average's nearness to Aristarchos' 1°/2 (reported by Archimedes in the same opus under examination), or [c] the wisdom of computing to 4 places using a 3 significant-digit number! (The same naïvete affected even Delambre 1817 [1:104], but that was back in an era when scientists were insensitive to significant-digits.) So Shapiro didn't calculate in reverse by simply (§E1) hypothesizing & checking to see whether 5400′/33′ rounded to 164 or to a different integer. He instead swiftly concluded by just echoing establishment dogma:

"the degree was, of course, not a unit used by Archimedes".

(If anyone among our readers knows of an earlier analyst who realized the sexagesimal truth behind Archimedes' solar brackets, please inform us so that we may add a citation here in this issue's next printing.)

E Archimedes' RtAngle-Unit-Fractions: His Solar Diameter Solved

E1 Since Archimedes' predecessors, Aristarchos (c.280 BC) & Aristyllos (c.260 BC), & likely Timocharis (c.290 BC) measured astronomical angles in degrees (see fn 3 & Rawlins 2008R fn 24), let us investigate eq.6 by the hypothesis that it expresses an empirical range, originally in degrees. Archimedes is our sole reliable witness to Aristarchos' solar diameter $d_\odot$, making it 1/720th of a circle or a half-degree (§C2; Archimedes p.223):

$$\text{Aristarchos:} \quad d_\odot = 30' \tag{7}$$

In eq.6, the number 164 is peculiar (prime factors: 2 & 2 & 41, which lead nowhere), so we are inspired to dig beneath the surface. Noting additionally that 200 and 164 differ by c.20%, we try the following hypothesis for explaining eq.6:

$$\text{Archimedes:} \quad d_\odot = 30' \pm 10\% = 30' \pm 3' \quad \text{or} \quad 27' < d_\odot < 33' \tag{8}$$

To test for confirmation we convert eq.8 into not only circle-fraction format (§D3) but specifically into satisfaction of another ancient schoolbook convention, *unit-fractions* (inverse-integers — as displayed in eq.6); we find the unit-fractions' denominators by dividing $27'$ & $33'$ successively into RtAng $= 5400'$, yielding 200 & $163\frac{7}{11}$, resp, which (after integralization of the latter) produces:

$$\text{Archimedes:} \quad \text{RtAng}/200 < d_\odot < \text{RtAng}/164 \tag{9}$$

— the very Archimedes bracket-expression (eq.6) we'd set out to trace the origin of. Being the least ambiguous of all entries in the list of evidences for 3rd century BC use of degrees, our finding now takes its place at the head of the list. And, with respect to Archimedes-as-astronomer (§D), eq.8 is absolutely accurate: the Sun never strayed outside limits $31'$-$33'$, so not only was his solar diameter correct but his brackets were judiciously applied.

E2 From fn 2, eq.8, & Rawlins 2008R fn 24, we see that, starting from dividing Earth C into 60 parts c.300 BC, Greek science transitioned to degrees early in the 3rd century BC.

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Answer[19] by *Alm* 4.6's method, there being "no question" (Duke 2008W p.286) he knew it.

K2 The one seeming "hit" here (presumably creating a sense of progress) is this: while Toomer 1973 p.15's 3134/338 got neither the Hipparchan e/R ratio (327 2/3 vs 3144) *nor* either of its factors, Duke 2005T p.170 by contrast nearly achieves all *simultaneously* by discerning (§K1) that changing only ζ_3 and by JUST (fn 19) the Right amount produces 3144 3/8 and 327 5/7, where the latter figure is taken as roundable to 327 2/3. (Would've been rounded to 327 3/4? Problemlet fixed at fn 19.) Such a match would lead any curious, able scholar onward to follow-up-investigate what initially smells like a faint hope of success. But such simultaneity's failure (§K4 below; Duke 2005T p.172) for Trio B suggests that Trio A's match was just coincidence, a likelihood enhanced by the admittedly (§K1) huge range of options by which one may manipulate the Trio B numbers until finally (eq.15) getting lucky. And enhanced further by realization that an utterly different explanation (Rawlins 1991W) solves both trios exactly. Note the stark essential contrast between Duke 2005T and the present analysis: all DR solutions involve ***ancient-typically round numbers, are independently***[20] ***supported*** and (unlike Duke 2005T p.172) ***equitably consistent for both trios*** (even the very same $g_\circ$ & $\epsilon_\circ$), ***while Occamly forsaking adducement of convenient Hipparchan miscomputations***. Summarizing: Aristarchos' famous 87° recovers $R = 3144$ (eq.5), which becomes 3122 1/2 (eq.6) via notoriously common misread (§C4); element-borrowing revealed by ultraclose $g_\circ$-agreement with 82° (§J; *and see especially* Rawlins 1991W §N10) for otherwise-jarring Trios A&B, **thereby ruling-out the 3-unknown Simultaneous Method**, which seeks $g_\circ$ as an unknown.

K3 An extra problem with the entire initial presumption that Trio A's 3144 & 327 2/3 materialized during a long mathematical juggle: were 3144 not fixed as R from the start, it would disappear through an obvious simplification; in a problem whose shaky input data (& discrepant e&r) would have suggested relative uncertainties of ordmag 10%, it would affect ratio R/r by barely 1 part in 1000 (ordmag 1/100th of ratio's uncertainty) to just round e to 328, then remove the factor 8 from numerator & denominator, leaving the neat ratio 393/41. This point reminds us again (as at §I5) that the proposed Toomeresque-processes for the two trios involve (§I10) so many arbitrary choices and cancellations, that it would be remarkable if the two R values agreed to within 1% by the accident implicitly proposed.

K4 For Trio B, a plainly unrelated number is again substituted for another (8;44,08 for Duke 2005T's $\delta_{\text{B3-B2}} = 8;46,28$), again of just the right size that ratio r/R comes out equal to Hipparchos'. But even this ploy won't produce Hipparchos' absolute values for r & R, as had been barely (fn 19) possible for Trio A; so, after this Trio B analysis gets as far as fixing the Right Ratio, a boldly arbitrary, Occam-defying *d-ex-machina* is brought in. A mere passing computation-facilitator (properly playing no rôle in final element-values of Ptolemy, Toomer 1973, or Duke 2005T's Trio A, as noted: p.172), final-fiddle-factor d is now for Trio B *ad-hoc*-manipulated by just arbitarily setting it at whatever (unattested) value neatly converts §K4-adjusted $r = 231$ 3/4 into 247 1/2 (p.172 step 5): namely, 3162.

K5 This also forces R to come out 3122 1/2 (step 8), since fudgery of $\delta_{\text{B3-B2}}$ (at §K4) had already been precisely designed to guarantee the process' issuance of the Right Ratio (12.616, as found unmanipulatively at Rawlins 1991W §N14); which, if simply multiplied times $r = 247$ 1/2, produces the desired attested (eq.4) $R = 3122$ 1/2. The astonishing[21]

[19] Beyond the 51° integral part, there's no resemblance between the former & latter (supposedly miscomputed) digits. (Thus the hemmed-in precision of the choice of substitute number is revealing: the only values that would produce a ratio roundable to 3144:327 2/3 are between 51;19,32 & 51;19,41. Actually, 50;19,33-36 would do better than Duke 2005T p.172's 50;19,37, since r would be more convincingly roundable to 327 2/3 than is 327.719 [likely to be rounded to 327 3/4], which follows from 50;19,37.)

[20] E.g., Rawlins 1991W §§K4&L3; *DIO 6* ‡3 §D6.

[21] We are asked to accept that Hipparchos' Trio B analysis specially used a π-value good to barely 2 decimal places — way worse than any known in competent antiquity (22/7, 377/120, Duke 2005T n.3, etc) — even in the very midst of the §I12 calculations, all of which are done to at least 4-place precision, **necessarily** using trig tables of like precision. See Toomer 1984 p.57 [n.68]; and note that

Toomer 1973 pp.11-12 reshuffles his diagram (admittedly falsely, supposing that Hipparchos made the same step inadvertently), and this time he gets (3082 2/3)/(246 1/3). Remarkably, considering Toomer has still missed by c.40 units a mark (3122 1/2) which Rawlins 1991W eq.24 was to hit *infinitely* more closely (above eq.6), Toomer concludes (emph added): "This is sufficiently close to Hipparchos' ratio [R/r = (3122 1/2)/(247 1/2)] to ***prove*** that Hipparchus did indeed use a chord table of the type [here] posited in computing it".

I15 A few years later, the foregoing Trio B development collapsed when an input datum was found (Toomer 1984 p.215 n.75) to be based upon a false reading of mss. (Computation using the correct reading led to an r of about 231 [Duke 2005T gets 231.727: see below at §K6] instead of 246 1/3. So Toomer's ***proof*** was a fantasy all along: *the imagined match was purely coincidental*.) To his credit, Toomer (at least temporarily) agreed that this correction "cast doubt" (*loc cit*) on his claim Hipparchos used a 3438-based chord table.

J Hipparchos' Overconsistent $g_\circ$ Reveals Aristarchos' $A_\circ$ = 96°

No one pushing Hipparchos' use of Ptolemy's Simultaneous Method (3 unknowns: §I1) ***uses it himself*** to find mean-anomaly-at-epoch $g_\circ$. *As Ptolemy always did* (*Almajest* 4.6), though *Alm* 4.11 cites Hipparchos' solution for **merely 1 unknown** for each trio, comparing his own lunar e (or r) to Hipparchos' but ***making no comparisons*** for the other 2 orbital data his 3-unknown method could find: $g_\circ$ & $\epsilon_\circ$. (Telling all but Muffiosi that Hipparchos neither sought nor cited either.) Three-unknown-solving for $g_\circ$ led DR to the entire trios-mystery's solution (Rawlins 1991W §§N4&N10): Trio A's $g_\circ \doteq 81°.8$; Trio B's, 82°.6. While quite inaccurate (real $g_\circ \doteq 87°$; *Alm*'s, 85°.3), the $g_\circ$ are near-equal (fn 9), shockingly so, given [i] $g_\circ$'s sensitivity to input-data uncertainty, & [ii] the big disparities of Trio A's e vs Trio B's r, and of the trio-analyses' underlying solar apogees (21° apart! *ibid* §K9vs§M4). These clues plus Rawlins 1991W §N10 are what suggested (*ibid* §§N4-N11) that $g_\circ$ was not sought by Hipparchos' math but rather was from a predecessor and thus set at 82°, so $A_\circ = 96°$ (*ibid* §N5 & eq.9; above eq.8) for both trios from the start. *Ibid* §§N4&N17 assign these (poor) values to Aristarchos. But Trio B's date could indicate lunar specialist Apollonios.

K Mythic-Centaurus Refereeing OKs Riggerous Mathematical Proof

K1 Nearly a third of a century after Toomer 1973, the journal *Centaurus*[17] published Duke 2005T, which attempted to salvage Toomer's theory by (for each trio, at a chosen step) changing one number to (***unlike above eq.5→eq.6***) an unrelated number, to MAKE said theory fit. For Trio A, presuming Hipparchos mis-computed ζ_3 (our $g_{A1} - g_{A3} + \delta_{A1\text{-}A3}$) as $51°19'37''$ (though accurate Duke 2005T calculation would yield $51°30'23''$) — openly stating no other justification but that this was necessary to get the Right Answer: Duke 2005T p.170. (Similar to Toomer: above at §I10.) Quoting *idem* on Trio A (emph added): "In order to get Hipparchus' answer we have to invoke some amount of rounding and miscalculation, so the first step is *to adjust something so that the correct numerical value* for the ratio R/e [3144/(327 2/3)] is produced. One simple way to accomplish this, out of an infinitude[18] of choices, is to assume that Hipparchus miscomputed [51;30,23 of *idem* p.169] as 51;19,37, but did everything else precisely. Then he would get" the Right

[17] Even beyond *Centaurus*' imperviousness to the (creditably undisguised) *ad-hok*iness of the proposed processes (esp. Trio B) — which led to *DIO* referee Hugh Thurston's rejection of them — there are printing problems here (none of which affect Duke's uniformly accurate calculations). These again (as we saw at R.Newton 1991 fn 7, Rawlins 1991W fn 126, Rawlins 1996C §B6) reveal hollow refereeing at *Centaurus*: [a] Nest of misprints in p.168's last paragraph (e.g., for $\alpha_3 = 360° - \alpha_1$ read $\alpha_3 = 360° - \alpha_1 - \alpha_2$). [b] At p.169 line 5, sign-typo; line 6, read 3135 1/7 for 3155 1/7. [c] Sign-slips in formula for R, at pp.172&173. (Harmless: Duke uses correct sign in actual calculations.)

[18] *Ibid* (pp.171&173) sees firm links between 3144&3438 and 247 1/2&3162; but each supposed link depends upon a specific choice of data-alteration among the cited infinitude of other possible options.

‡2 Ancient Solstices

Ancient Solstice-Determiners' Delicate Voyage 'Twixt Random Error's Scylla and Systematic's Charybdis

Tihon Finds Hipparchos' −157/6/26 18$^{\mathrm{h}}$ Solstice Its Significance and Neat Surprise-Solution

New Light on Hipparchos' Calendar, Solar Elements, & Year-Length

A Summary & Unwelcome Shock-Confirmation of *DIO* Prescience

In 2010, Anne Tihon (www.springer.com/us/book/9789048127870) meticulously analysed a recently recognized papyrus (*P.Fouad* 267A, fortunately recommended to her expert examination by Jean-Luc Fournet) bearing: Hipparchos' −157/6/26 Summer Solstice, use of his hitherto-unknown 500$^{\mathrm{y}}$ solar longitude tables (one of them Kallippic), a new precession rate for the tropical points, also a new ancient yearlength which D.Duke soon correctly reasoned was based on comparison to Meton's −431 S.Solstice. Below, we show how ancient solstices were determined outdoors — as well as detailing the problems Hellenistic scientists had to balance, to achieve an accurate estimate of a solstice's hour. We also examine why the best ancient scientists preferred solstices to equinoxes as bedrocks for their calendars; and we consider the newly-available −157 solstice's implications for dating some of Hipparchos' astronomy. Curiously, no commentator on the papyrus' −157 solstice has yet remarked that the 1$^{\mathrm{st}}$ and only prior paper to propose (§K) Hipparchos sought a −157 solstice & used Kallippic mean solar motion is Rawlins 1991W. Do non-citers believe *DIO* happened only by blind luck to improbably [a] hit upon the now-papyrus-confirmed date of Hipparchos' 1$^{\mathrm{st}}$ try at a solstice (& orbit), [b] induce his Kallippic solar speed?!

B *Journal for the History of Astronomy* Biggies' 4 Solstice Adventures Sending History-of-Astron's MacArthur Genius Up to 9$^{\mathrm{th}}$ Grade

The laugh-crying need for a competent article on mathematical and historical matters regarding ancient solstices may perhaps be brought home to the reader by a swift foray here into the wisdom on the subject that's been emanating from academe's two most highly-placed and expensive Experts[1] in the field of ancient astronomy. [It would not be necessary to highlight the weird stuff that follows here, except that — typically for cohesive, wagon-circling cults — despite years of opportunity (and *DIO* nudges), the perps ***have not retracted on-the-record a single one of the strange-science adventures we enumerate below***.]

B1 A.Jones, sometime Princetitutee, now at NYU's hugely endowed Inst. for the Study of the Ancient World, Boardmember-for-Life at history of astronomy's "premier" (Schaefer 2002 p.40) *Journal for the History of Astronomy* (& *JHA*'s discoverer of the Winter Equinox: Jones 1991H p.119), has added to *JHA*'s rep (www.dioi.org/jha.htm#kqlz) for meticulous refereeing by rejecting in its pages the reliable standard ancient method (*Almajest* 1.12) for finding latitude&obliquity via solstices, using equinoxes instead: Jones 2002E, a paper taken rather too seriously by PU's history of early trig, van Brummelen 2009, p.65 n.76, though with fair citation of *DIO 4.2* p.56's [or Rawlins 2009S p.20's] stark Table 1. (Unlike Jones, who persists in nonciting this Diller-DR table's *perfect* data-fit, to fake Jones 2002E's viability, not even producing his own table! *Do not miss fn 10 below*.)

[1] Despite their here-appreciated screwball gags, each of our roastees has made solid contributions to knowledge, as seen at, e.g., §B4, *DIO 4.3* ‡13 §D8, *DIO 11.2* cover [owed to Duke&Jones], *DIO 12* ‡2.

B2 Noel Swerdlow (Yale, UChi, CalTech), History-of-astronomy's MacArthur-Genius & *Journal for the History of Astronomy* Boardmember-for-Life, has, like *JHA* Assoc.Ed. J.Evans 1998 p.206, spent decades misunderstanding the 9th-grade-level method used by ancients to measure solstices, an achievement recognized by DR at R.Newton 1991 fn 20:

> One of the more amusing moments in [HamSwerdlow 1981], which RRN is too polite to note, is [HamSwerdlow 1981]'s sarcastic mock astonishment while commenting upon a key RRN discrimination: "most remarkable of all, that solstices could be observed with more accuracy than equinoxes." That RRN is correct (in the very judgement which *HS* attack as "remarkable" folly) is obvious to any unprejudiced scientist familiar with the instrumental problems involved. (See the lucid discussion at R.Newton 1977 pp.81-82 [or §G1 here].) . . . *all* known ancient astronomical-observer-calendarists (excluding [indoor] Ptolemy . . .) depended primarily upon solstices for gauging the year's length: Meton, Euktemon, Kallippos, Aristarchos, Hipparchos. (Hipparchos observed numerous equinoxes [§O]; but even *his* year-lengths were based upon solstices: see, e.g., [Rawlins 1991H] eq.8 [& below eqs.32&34].) However, Swerdlow, an historian [then] with the official rank of professor [at U.Chicago's Astron.Dep't] cannot understand this elementary point: during a gloriously delirious passage (p.527) in his prominent 1979 attack on Newton (in *American Scholar* [Phi Beta Kappa!] *48*:523 . . .), Swerdlow argues:
>
> > At the time of the solstice, the meridian altitude of the sun changes by less than fourteen seconds of arc per day, and measuring this quantity, let alone any fraction of it, was obviously ridiculous.
>
> The only ridiculous aspect of this astounding piece of reasoning is that a member of *the University of Chicago's Dep't of Astronomy* should so conspicuously exhibit his touching innocence of the implications of 1st-year calculus and of the standard technique known[2] as "equal altitudes". It is easy to see that Hist.sci archon Swerdlow's reasoning is essentially equivalent to insisting that the time a vertically oscillating body reaches maximum altitude cannot be determined since at that moment it lacks vertical motion!

Or, to reduce this to around junior-high: our nay-jerk R.Newton-hater (*DIO 1.1* ‡3 §§D2-D3) is essentially claiming that if you toss a ball upwards at t_1 and catch it at t_2, it is "ridiculous" to suppose that its height maxed at $(t_2 + t_1)/2$.

B3 Far from admitting his elementary misunderstanding, invincibly-ineducable Swerdlow keeps promoting the same reasoning's validity a decade later,[3] in the very *Journal for the History of Astronomy* paper (Swerdlow 1989 p.36) which got him his MacArthur!

[2] The Bowditch *American Practical Navigator* 1981 ed. 2:799 defines equal altitudes thusly: "Two altitudes numerically the same. The expression applies particularly to the practice, essentially obsolete, of determining the instant of local apparent noon by observing the altitude of the sun a short time before it reaches the meridian and again at the same altitude after transit, the time of local apparent noon being midway between the times of the two observations, if the second is corrected as necessary for the run of the ship. [*DIO*: And solar δ-shift.] Also called DOUBLE ALTITUDES." See fn 6 here.

[3] Rawlins 2002V fn 20: "In this MacArthur-grant-subsidized paper (published by Gingerich's *JHA*), [Swerdlow 1989 p.36] . . . alibis that since (near maximum) Venus' elongation changes merely $1^\circ/12$ in 6^d, 'in no way could Ptolemy estimate the time' of greatest [maximum] elongation more accurately. (Gingerich 2002's incomparable p.72 goes even further into legalblindnessland, claiming that ***one-degree***-accuracy in observation 'is what Ptolemy typically worked with' — a sleight which neatly confounds ordmag $0^\circ.1$ ancient observational accuracy [§B4 & Rawlins 2009E] with the ordmag 1° enormity of the most delicious Ptolemy fudge.) We have already previously ([R.Newton 1991] fn 20) dealt with the tragic pre-highschool mentalblindnessland adventure of Swerdlow 1979 pp.526-527 (in the journal of PhiBetaKappa), regarding estimation of maxima-times (solstices in that case), so I won't reprise the pathetic details here merely because he later repeated the folly under the MacArthur Foundation's aegis. But . . . none of this excuses inaccuracies of several weeks in Venus observations, leading to *dishonestly-reported* 'observational' [V elongations] which are off by way over *a degree*."

I10 The point becomes particularly relevant when we notice that the §I8 line (step 5) which produces $R = 3134/3344$ (in units of s): came from dividing numerator 6268 & denominator 6688 each by 2. But: why not divide by 2 again?! [Duke 2005T p.171 passes over same point when p.170 line 6 conjures-up 3144.] Why doesn't Toomer simplify 6268/6688 to 1567/1672 instead of 3134/3344? (Previous line: exact calculation produces 6270/3438, so equitably dividing by common factors 2&3 produces 1045/573.) Not remarking the plain divisibility of 6268 & 6688 by 4, Toomer 1973 p.27 n.14 just says (emph added): "I suppose division by the common factor 2 here. It must have occurred at some stage *in order to get Hipparchus' final result*." (Same circular reasoning below: §§I14&K1.) I.e., ***prove Hipparchos used the 3-unknown method by assuming he did***.

I11 The foregoing §I10 failure-to-divide-by-2 revelation is just another example of an earlier point (§I5) regarding Trio B: the very r/R ratio being sought, (247 1/2)/(3122 1/2), would have immediately (after factors' division by 5/2) become 99/1249. Similarly for Trio A's r/R ratio: (327 2/3)/3144 → 983/9432. Why carry fractions in a fraction unless something fundamental is being held fixed? So it makes more sense to suppose that Hipparchos had adopted his R values (3144 & 3122 1/2) before his mathematical searches for e & r even started.

I12 Toomer 1973 pp.10-11 Trio B successive line-segments (units of s, except last line):

$$\frac{1000}{6669\ 1/3}$$

$$\frac{1112\ 1/2}{6750\ 1/2}$$

$$\frac{2989 \cdot 1112\ 1/2}{2 \cdot 3438 \cdot 6750\ 1/2} = \frac{246\ 1/3}{3438}$$

$$\frac{5\ 1/3}{3438}$$

$$\frac{246\ 1/3}{3438}$$

$$r = \frac{3438 \cdot 246\ 1/3}{2989 \cdot 3438} = \frac{246\ 1/3}{2989}$$

$$\frac{2372 \cdot 246\ 1/3}{2989 \cdot 3438}$$

$$R/r = \frac{2913}{246\ 1/3}$$

I13 When 246 1/3 (close to Hipparchos' $r = 247\ 1/2$) appears (in §I12's step 3), it is not multiplied or divided in the steps that follow, thereby ensuring its suspended-animation-survival for ultimate display in a ratio for comparison to Hipparchos'. So the near-grail of 246 1/3 is captured only due to the *choice* to set aside and retain 3438 in the 3rd line of the above, while simultaneously *merging* four other numbers:

$$\frac{2989 \cdot 1112\ 1/2}{2 \cdot 6750\ 1/2} = 246\ 1/3 \tag{14}$$

(Exact calculation from the outset yields 2990 & $r = 246\ 1/2$, and $R = 2918$ or 3079; the former from a non-fudged diagram; the latter, from allowing Toomer's below [§I14] re-draw of it.) Toomer 1973 p.11 realizes that 2913 isn't near the desired $R = 3122\ 1/2$. So, does this show that that the theory is disconfirmed? No, it continues (Rawlins 1991W §P1) in high regard at hist.astron's political center.

I14 Undeterred by 2913's remoteness from 3122, Toomer&co have attempted to FORCE this by-now Muffia-sacred method to work. Toomer 1973 was first to do so, proposing (p.11) a weird error of method. (Which still failed to find good matches.) His reasoning: if the number doesn't match, "we must . . . suppose that [Hipparchus] made a mis-calculation."

fn 244): 3144 & 3122 1/2 *are within 1% of each other* — a central point ***never even noticed*** by those promoting disparate on-the-fly origins for these numbers, thereby implying the super-proximity is just an amazing accident, (§K3), instead of wondering whether it was symptomatic of a common origin, which has turned out to be the now-obvious truth (§C4).

I6 To effect his midstream-fallout version of the Simultaneous Method, Toomer follows the same calculation-procedure as *Almajest* 4.6. He starts by drawing a line from the Earth (point O in Toomer 1973's diagrams) through any one of the three eclipse positions (points M_i for i=1-to-3 or I-to-III, in his diagrams), calling B its other intersection-pt with the lunar orbit — and calling "s" the line-segment from O to B, a device that permits geometry to solve the problem. (In Duke 2005T's revisitation of Toomer 1973, s is called d.)

I7 Toomer adds to the speculativeness of his conception by supposing that Hipparchos did not use the Greek chord (crd) table of *Almajest* 1.11, where values range from 0 to 120 (angles 0°-to-180°):

$$\text{Greek:} \qquad \text{crd}\,\alpha = 120\sin(\alpha/2) \tag{12}$$

— but instead for each trig calculation took a number c.180/π times bigger:

$$\text{Indian:} \qquad \text{crd}\,\alpha = 6875\sin(\alpha/2) \tag{13}$$

I.e., Indian tables' values range between 0 and 6875, being based upon a circle-radius of 3438 units.[16] This ensures that each trig calculation is likely to produce numbers in the thousands. As each Toomer trio-calculation proceeds, one of these numbers of course may happen to hit near one of the two he's looking for (either will do). (If neither desideratum turns up, he can try the same calculation by drawing line s through either [§I6] of the other two eclipses of the trio and computing on that basis. [See Toomer 1973 p.27 n.14.] During each trio's Simultaneous Method analysis, $R' = 3438$ enters at step 3 due to the method's dropped-perpendicular ploy [*Alm* 4.6].) Let's watch it happen, starting with Trio A.

I8 Toomer 1973 pp.13-14 Trio A successive line-segments (units of s, except last line):

$$\frac{3110\ 1/2}{6574}$$

$$\frac{5379}{3247}$$

$$\frac{5379\cdot 6688}{3247\cdot 2\cdot 3438}$$

$$\frac{6268}{3438}$$

$$R = \frac{6268\cdot 3438}{3438\cdot 6688} = \frac{6268}{6688} = \frac{3134}{3344}\ \text{[why not} = \frac{1567}{1672}\text{? — see §I10]}$$

$$\frac{3134\cdot 6853}{3344\cdot 3438}$$

$$R/e = \frac{3134}{338}$$

(Exact computation all the way through would instead produce 3135/335 = 627/67.)

I9 Given that Toomer calculated his method from three different starting points (Toomer 1973 p.27 n.14), it is no great shock that somewhere along the way a number (3134 at §I8's step 5) indeed pops up that's near one of the Trio A sought-for R (3144) or e (327 2/3). The only genuine surprise here occurs when Toomer supposes that Hipparchos would at this point suddenly and selectively preserve the number 3134 all the way to the end of the calculation. (One can understand Toomer freezing such a number, since nearby 3144 is one of his goals. But why would Hipparchos care about packing this value in ice?)

[16] There being 21600 arcmin in a circle, the consistent radius is that number divided by 2π: slightly less than 3437 3/4, the number of arcmin in a radian. The Indian table proposed for Hipparchos (Toomer 1973 p.8, Duke 2005T p.175) is effectively the Ptolemy (*Almajest* 1.11) table at 7°1/2 intervals (see also ‡1 fn 1) — with each Ptolemy chord-value enhanced by factor 180/π and integrally rounded (4-place precision). See Neugebauer 1975 pp.299-300, 319, 1116, & p.1132 Table 8.

Such obstinate-incredible semi-numerate escapades by ultra-archons, serve the useful purpose (additionally to funnybone-exercise) of measuring the hist.astron community's relative skills at science vs six-figure-profitable careerist politics. See similarly at Rawlins 2009S. Dozens more such Premier-journal larfs are chronicled at www.dioi.org/jha.htm#lmvl.

B4 [Section added 2012/12/29.] *JHA*'s 2008 Aug Pb paper Duke 2008W is yet another snooze-refereed solstice-study, claiming Greek solar-declination δ data had *random* error of standard deviation $\sigma_\delta = 15'$ (vs actual c.1′: eq.14, below), citing irrelevant (*DIO 16* ‡3 fn 36) Greek star $\sigma_\delta = 10'$ (vs real median 5′: *DIO 4.1* ‡3 Table 3), & speculating Greeks found solstices via vast melds of motley-weight equal-altitude pairs with ere&aft intervals d ranging from 20^d to 55^d; result unquantified beyond meld's $\sigma_{SS} = 5^h$. But ancient Greek $d = 55^d$ would cause S.Solstice systematic error -8^h. If σ_δ really were $15'$ (nearly the span from solar center to limb!), random error would've been half a day for $d = 55^d$ and *over a day* for $d = 20^d$. Actual Greek solstices' errors: 0^h-3^h (§§H-J & Table 3). Hipparchos' Rhodos 147-128BC equinox data (excellent list Duke 2008W Table 1): *systematic* δ error $+6'.5\pm0'.4$ [pioneer John Britton 1967 p.24 got 7′; R.Newton 1977 p.78 same] (7^h-), accounted-for (within c.1′) by $+4'+$ correction to noon altitude h for Hipparchos' fantasy 7′ solar parallax (Swerdlow's neat discovery: Rawlins 1991W fn 280), plus non-correction for sunlight's 0′.7 atmospheric refraction, plus transit-circle 1′1/3 mis-set via refracted polestar-light ($\sigma \doteq 1'$, like Hipparchos' geographical latitude σ from stellar δ data: Rawlins 1994L Table 3); *random* error $\sigma_\delta = 1'3/4$, near-same as rms 1′.7 scatter from 1/4-day rounding (constraining raw empirical random error to c.1′). Rounding cardinal-point solar data so was calendaric-tradition and-or Aristyllan-modest overcaution (‡1 §C1; *Alm* 3.1) against being responsible for unreliable data. (Note: Hipparchos' 162-158BC equinoxes' systematic error may've been mostly from asymmetric gnomon.)

C Precisely Determined Ancient Solstices

C1 Given the hist.astron center's continuing problems in the area of solstices (e.g., Rawlins 2009S §F3), it will help if we cite (and later list: Table 3) what we have hitherto possessed of outdoor ancient solstices where *the hour not merely the date is known*. After discounting those (Table 1) truncated to day-epoch — Meton's (−431) & Aristarchos' (−279) and the faked solst (Table 2) of Ptolemy (+140) — we find that we have just four so far, most only by modern reconstruction, not direct attestation. (The exception is −146, confirmed by *P.Fouad* 267A: §M4.) The −329 S.Solstice launching Kallippos' famous calendar is reconstructable by realizing (§J4; Rawlins 1985H) that his pioneering yearlength (nearly 3 centuries before Julius Caesar's Sosigenes), $Y_K = 365^d1/4$, arose from his comparison of his own S.Solstice observation to Meton's famous Athens −431/6/27 S.Solstice, which was typically (for calendarists) truncated to the beginning (sunset for Athens) *of the* 24^h *period containing the event*. So add $102Y_K$ or $37255^d1/2$ to the start of Meton's calendar to find the solstitial moment of the Kallippic calendar's start:

$$-431/6/27\ 3/4 + 102\cdot(365^d 1/4) = -329/6/28\ 1/4 \tag{1}$$

($+3^h$ error). Like logic allows reconstruction of Aristarchos' −279 solstitial observation, using his Saros-cycle-fitting (Rawlins 2002A fn 14; Rawlins 2018C §G6) yearlength, Y_A to go 152^y (8 Metonic cycles) beyond Meton:

$$-431/6/27\ 3/4 + 152\cdot(365^d 1/4 - 15/4868) \doteq -279/6/27\ 1/4 \tag{2}$$

(0^h error), which was truncated to −279/6/26 1/2 for the Dionysios calendar's day-epoch, which was *noon* (Table 1), as 1st computed by Rawlins 1985H (& Rawlins 1991H eq.8). Similarly, starting with Babylonian Astronomical Cuneiform Text 210 (BM55555), whose Greek-based year-length is

$$Y_{U\text{-}M} = 365^d 14' 44'' 51''' \doteq 365^d 73/297, \tag{3}$$

we know from Rawlins 1991H that Hipparchos' 135BC solstice (which he used to found his final "UH" solar orbit: *ibid* §C) occurred 297 of ACT 210's years $Y_{U\text{-}M}$ or 108478^d (§P6

Table 1: Ancient Calendarists' Truncated Solstices

Solstice Observer	Truncated Time	Real Time	Error
Meton	−431/6/27 3/4	6/28 11^{h}	-17^{h}
Aristarchos	−279/6/26 1/2	6/27 06^{h}	-18^{h}

Table 2: Ancient Astrologers' Indoor-Calculated Solstices

Solstice Calculator	Computed Time	Real Time	Error
Hipparchos	−157/6/28 1/4	6/26 18^{h}	$+36^{h}$
Hipparchos	−157/6/26 3/4	6/26 18^{h}	$+00^{h}$
Ptolemy	140/6/25 1/12	6/23 14^{h}	$+36^{h}$

Table 3: Ancient Astronomers' Firm & Precise Outdoor Solstices

Solstice Observer	Observed Time	Real Time	O−C Error
Kallippos	−329/6/28 1/4	3^{h}	$+3^{h}$
Aristarchos	−279/6/27 1/4	6^{h}	0^{h}
Hipparchos	−146/6/26 1/2	10^{h}	$+2^{h}$
Hipparchos	−134/6/26 1/4	7^{h}	-1^{h}

below) after Meton's Solstice as misunderstood by Hipparchos. (Who interpreted Meton's start-of-day as dawn instead of Athenian sunset. Perhaps just to find or force a fit to the overlong Metonic lunisolar scheme? See below at §§P4-P5 & §Q1.) Rawlins 1991H eq.6:

$$-431/6/27\ 1/4 + 297 \cdot (365^{d}73/297) = -134/6/26\ 1/4 \qquad (4)$$

Zzzzz-reffed Duke 2008W Table 1 crucially (‡3 fn 8) misclaims *Almajest* 3.1 makes it noon.

C2 Incredibly, Kallippos', Aristarchos', & Hipparchos' reliable, precious, professionally-observed solstices' precise hours have never [been perceived outside of *DIO*] — much less deservedly highlighted at last by exclusive tabulation. We do the honors here (Table 3), as we discover a hitherto-unknown addition (eq.27) to the list.

D Hellenistic Astronomers' Outdoor Empiricism

The three accurate solstices cited (eqs.1-2&4) add to the accumulated evidence that Greek astronomers were anything but the dreamy, data-inventing critters that certain truly dreamy historians imagine. See, e.g., our comments (at Rawlins 2008R §A) on Muffia god-pop O.Neugebauer's strange vision. Other evidences of Greek empiricism's accuracy & primacy (Rawlins 2008Q §K4 & n.9) include the half-percent precision of Greeks' basis-measure for finding the Earth's radius (‡1 §B3) — and more spectacularly their three lunar periods (‡3 fn 27; www.dioi.org/thr.htm), ***each accurate to better than 1 part in a million***.

E Truncated Solstices

We list all extant day-start-truncated solstices (§§C1-C2&E1-E3) in Table 1.

E1 Meton's calendar started on −431/6/27 3/4 since Athens' day began at sunset. As late as a century after, Kallippos knew the original Meton calendar epoch and (eq.1) founded his year-length upon it — though (§§C1&P4) Hipparchos later misconstrued Meton's S.Solstice by -12^{h}, making its error -29^{h}, which caused (along with eq.31) huge systematic errors in later astronomers' yearlength estimates (§Q1; Rawlins 1999 §B6).

R from the process, they neglect $g_{\circ}$ & $\epsilon_{\circ}$ — omissions we have not repeated: Rawlins 1991W eq.9 & here at §J & eq.8.) Comments: [a] All such studies forget the possibility that Hipparchos cites trios not because of his math but simply because, at any given place on Earth, lunar eclipses can naturally occur in threes, during a brief period: less than a year. (Though: see Rawlins 1996C fn 103 for two unusual tightquads: −831-830 & −720-719.) [b] Simple pair-analysis produces perfect hits (Rawlins 1991W §P2) on the Hipparchan e & r (327 2/3 & 247 1/2) sought for Trios A&B. (See arrowed data in §§G2&F2, resp.) Meanwhile — unless (§K1) the calculations are flagrantly fiddled to produce the desired result — none of the centrists' complex trio-math attempted solutions has ever reproduced either (much less both) of these same Hipparchan numbers.

H3 Nor has the untampered Toomer fallout-in-midmath (§I9) reverie hit upon either of Hipparchos' R values (3144 & 3122 1/2), while Rawlins 1991W's eqs.23&24 showed that one line (here at eq.5) of Aristarchos-based trig leads to both, on-the-nose: above, §§C2-C4.

H4 But hist.astron's political center insists (§A2; Rawlins 1991W §§D4&H2) upon preferring (some variant of) its own cult's 4 decade-old solution. No hist.astron-centrist scholar has ***ever*** (e.g., fn 22) let its readers in on the fact that all 4 of these numbers were (above §H2 [b] & §H3) *precisely* matched by *ibid*. Compare to the *history of science* center — which a decade ago courageously defied the *JHA*-Muffia cult by disseminating Rawlins 1991W's results in the world's leading history of science journal, *Isis* (Thurston 2002S). [In 2016, *Isis* lost 2002 Editor Margaret Rossiter's openness: see letter Rawlins 2018C.]

I Undead Fantasy: NonExistent 3438-Radius Greek Chord Table

I1 The impasse here is based on a fundamental Muffia misunderstanding that takes for granted ("virtually certain" [Duke 2005A p.5]) that Hipparchos' analyses of the 2 trios were by the elaborate method Ptolemy develops so ably at *Almajest* 4.6: from 3 eclipses' times, solve for 3 planar lunar orbital elements simultaneously, thus here the "Simultaneous Method" (§H2).

I2 Toomer 1973 claimed to have shown that Hipparchos used the Simultaneous Method, doing his trig by a chord table based upon circle-radius $R' = 3438$, which is the number of arcmin in a radian. Such tables were used the better part of a millennium later in India, which drew some astronomy from the Hellenistic tradition, thus the superficial [Greek army left India 325 BC, two centuries before Hipparchos] plausibility of the Muffia's refreshingly original idea. (An inspiration which must have greatly pleased Indian-astronomy specialist and fellow-BrownU scholar & mentor David Pingree.)

I3 But there is not-a-shred of direct evidence that Hipparchos ever used such an odd device as a 3438-based trig table. Thus, Toomeresque theorizing has managed to pioneer not-a-shredness-squared here vis-à-vis Greek astronomy: R being found on-the-fly (see below: §I9), plus an Indian trig table flourishing centuries before its earliest attestation. And the Muffia claims to detest speculation by outlanders.

I4 What drew Toomer into his theory was the crude proximity of 3438 to both Hipparchan lunar R values: 3144 & 3122 1/2. When he applied the Simultaneous Method to Hipparchos' data (3 distinct ways [§I9] for each trio): during the cascading stages & mergings of the attempted 3438-based computational reconstruction, there inevitably appeared somewhere (anywhere would do) numbers that roughly approximated these two Hipparchan elements — though none came convincingly close to actually matching them.

I5 The most immediately obvious clue that Toomer — though admirably masterful at the geometry involved — is pursuing a chimera is: if ratio 247 1/2 vs 3122 1/2 had not been based upon a start-out presumption of lunar orbit-radius 3122 1/2 (as shown above at eq.6) why would the ratio not be converted by Hipparchos to 99/1249, just like the conversion (below) of eq.19 to eq.18? (Even the same conversion-factor: 5/2.) This alone (and see analogously below at §§I11&K3) tells us that lunar-mean-distance radius 3122 1/2 was adopted BEFORE not DURING Hipparchos' calculation — a priority which is consistent with all known Greek astronomical work. Further indication (as noted at Rawlins 1991W

Unfortunately, Hipparchos chose the 2nd solution (arrowed A3-A2) for publication, a solution *squarely based upon fabrication*. Its e (which of course would be rounded to 327′2/3) agrees with the attested Hipparchan value: eq.3. [Note added 2012/9/16. For both e&r, he picked the value *nearest a round fraction* (327 2/3 = 6p1/4, 247 1/2 = 4p3/4): which backs *DIO*'s theory (e.g., eq.8, fn 11, Rawlins 2002A §A6) that ancient astronomers preferred round numbers for elements. (And for observational data: R.Newton 1977 pp.250-254. Newton's discovery clinched by 30′ endings: *DIO* 2.3 ‡8 fn 47 & Rawlins 1994L fn 5.)]

G3 Notice that we now not only have the why of fabrication but additionally have shown that it arises out of the theory that Hipparchos was mathematically investigating the lunar orbit via pairs not trios. The foregoing is therefore a surprise vindication — unanticipated in Rawlins 1991W — of *idem*'s Pair-Method explanation of the curiously skimpy (& explicitly-in-pairs) early data which Hipparchos&Ptolemy left us via *Almajest* 4.11.

G4 Check the 179° difference between Hipparchos' pre-existing (eclipse-catalog: §E2) Frankenstein-orbit-computed *solar* $\phi_{\mathrm{A3}} = 257°7/8$ (Rawlins 1991W §M10) and the subsequently −1°-fudged *lunar* $\phi_{\mathrm{A3}} = 76°7/8$ (*ibid* §N15) — a forgery which naturally also shifted contingent $\delta_{\mathrm{A3\text{-}A2}}$&$\delta_{\mathrm{A1\text{-}A3}}$ from their §E4 values to the 1°-altered values of §G1, which underlie §G2. [Hipparchos didn't recompute (§B1) his eclipse catalog's solar ϕ_{A3} (to check vs lunar ϕ_{A3}) or spot R's §C4 devolution.] The 1° discrepancy has been known at least since R.Newton 1977 p.119's clear explanation of the joke solar-speed which non-correction entails. (See also Hugh Thurston 2002S p.67. Duke 2005T p.176-177 n.5 ignores R.Newton, DR, & Thurston, though doing so results in a wild solar-orbit eccentricity[14] of over ***thirteen percent*** as 1st noted at Rawlins 1991W fn 162 [$e \doteq 8/60$], & [non-citationally] agreed to at Duke 2005T p.177 n.5.) But no one (incl. Rawlins 1991W) previously realized that the error was due to a deliberate shift — to forge an orbital fit. Frankensteinorbit (§E2) is obviously & variously superior (to a 13%-eccentric EH solar orbit!) — since its apogee-at-epoch $A_\circ$ (PH) and eccentricity[15] e (EH) are much nearer reality (e.g., eccentricity 5% vs then-actually 4%) than the 13%-eccentric monstrosity required if 1°-fudge isn't corrected-for. Concluding this section: We have established Hipparchos' adoption of a one-of-a-kind (and physically impossible) celestial configuration: a 179° difference of solar true longitude vs lunar true longitude for −381/12/12 mid-eclipse — adopted to paper over problems with his eclipse researches. In astronomers' terminology: a mid-eclipse Moon at 179° true elongation — Hipparchos' astonishing 1° fake, which has now (§G1) for the 1st time been fully solved, and thereby detected as fraud.

H Centrists & Rebels

H1 Several discussions of the Hipparchos trios have appeared in recent decades out of the history-of-astronomy center, e.g., Toomer 1973, Neugebauer 1975 pp.315-319, Jones 1991H, etc — along with two rebel studies, R.Newton 1977 p.115-129, and Rawlins 1991W. The former authors all take the data as entirely real; the latter dissent (in differing fashions).

H2 Contra Rawlins 1991W, centrist authors propose (or accept) that, for each trio, Hipparchos' analysis found unknowns *simultaneously* from his three time-data. It would indeed be possible thus to find three lunar-orbit unknowns; mean-longitude-at-epoch $\epsilon_\circ$ (in degrees), mean-anomaly-at-epoch $g_\circ$ (also in degrees); and eccentricity e (Trio A) or epicycle-radius r (Trio B), either expressed in 60ths of a unit. But the centrist studies instead attempt to apply a newly (and wholly) invented Hunt&Freeze technique to go beyond what used to be the limit for three equations of condition, to try pulling from the data four unknowns — adding R to the hunt. (Though in their determination to conjure e [or r] and

[14] Remember that what ancients (using adjusted circular orbits) called eccentricity was twice what moderns (using elliptical orbits) refer to by the same term. Rawlins 1991W fn 162 found that if eclipse A3's −1° fudge is not accounted-for (i.e., undone), the data are consistent with $e = 7^{\mathrm{p}}46'$ or 12.9 percent ancient convention; 6.5 percent, modern.

[15] Again (fn 14): these e values are ancient-convention. Modern equivalents would be half as large.

E2 Exactly 2 Kallippic cycles after Meton's S.Solst & less than 3d before the −279/6/30 total lunar eclipse, Aristarchos' *observed* −279 S.Solstice (eq.2), as distinguished from his *calendaric* S.Solst, may've helped retro-firmup establishment of heliocentrists' epoch −284 Dionysios calendar (§C1, 1st reliably reconstructed by Böckh 1863 & van der Waerden 1984-5), which used Kallippos' yearlength Y_{K}), and generically differed little from his calendar, maybe adding embedment of Aristarchos' Great Year (Rawlins 2002A §A4).

E3 Aristarchos' day-epoch-truncated calendaric S.Solstice (Rawlins 1991H eq.8) is reconstructable from Hipparchos' −134 solstice, combined with the *Almajest* 3.1-attested interval. (See Rawlins 1991H eq.8.) Thus, in Table 1, each of the truncated solstices was later used by Hipparchos to find the length of the year, where the truncations contributed to results that were seriously too long — below eqs.32&34 — but (§P7) just about right for matching Metonic preconception's eq.31 (perhaps based on luni-solar politics: §P8).

F Equal-Altitudes: How the Ancients Determined Solstices

F1 As noted at §B2, *DIO* has for decades asserted (against Muffia-MacArthur genius-dum) that ancient solstices were observed via Equal-Altitudes. Understanding the method shouldn't challenge a high-schooler.

F2 Starting d days before the Solstice, as the Sun transits (culminates) at Local Apparent Noon (LAN), the observing astronomer records in degrees and arcminutes[4] the altitude h of the Sun's center (preferably just a few degrees below the h_{SS} of eventual solstitial culmination). This noon will be called t_1. By obvious symmetry, the LAN Sun's altitude will be back near[5] h at d days after Solstice, LAN-culminating at a time which will be called t_2. The midpoint between the two times is then taken as the Solstice-hour t_{SS}:

$$t_{\mathrm{SS}} = \frac{t_1 + t_2}{2} \tag{5}$$

Obviously, the 2 times' relation to d is (see further at §§G3&§§J1-J2 & eqs.19-21):

$$d = \frac{t_2 - t_1}{2} \tag{6}$$

[Meaning pair-means for several d (e.g., 19d, 20d, 21d) can ensure reliably accurate solst.]

F3 The method is attractively[6] simple. But the Equal-Altitudes Method is subject to small errors (to be quantified below: §§H-I), which had to be carefully accounted for, by any ancient scientist intending to acquire maximally accurate naked-eye results using it.

G Solstice-Observation Technique: Going Beyond Naïve Eq.5

G1 The great accuracy-advantage of solstices vs equinoxes is this: if there is uncertainty in adopted solar parallax, atmospheric refraction, the transit-instrument's mounting or secular settling or arc-ruling-uniformity, then an equinox-timing is corrupted (§B4) by each's systematic error. But not a solstice, since all these errors' effects on t_1 & t_2 are nearly the same while of opposite sign, thus leaving eq.5 unaffected. Yet solstitial determination has its own problems, which are [a] lesser, but [b] serious and (except for random-error problems, which are smaller with equinoxes) completely different from the traditional bothers for equinox-observations.

[4] The ultimate new proof, that Hellenistic scientists had adopted Babylon's sexagesimal measure for angles as early as the 3rd century BC, is found here at ‡1: Archimedes' masked solar diameter brackets.

[5] Astronomers will see that we are merely using h to measure solar declination δ, in order to find two times on either side of Solstice when δ is the same. Generally, finding the exact time t_2 when the 2nd estimate of δ exactly matches (that which occurred at t_1) will require interpolation — since only by rare luck does the post-Solstice h_2 match the earlier noon h_1 almost exactly at noon.

[6] Our present annual version of the technique has a diurnal parallel (fn 2) often used by pre-GPS-era explorers. (Among others: the Isaac Hayes & Rob't Peary expeditions.) Secondary-school classes teach an analogous method for finding when a thrown ball reaches maximum height: §B2.

G2 If the Sun's motion were uniform, it is obvious from symmetry that LAN solar altitudes h_1 & h_2 measured at respective times t_1 & t_2 with an instrument set at constant solar altitude h so that

$$h_2 = h_1 \tag{7}$$

would ensure that the corresponding solar longitudes are each at the same angular distance S from the S.Solst point:

$$90^\circ - \phi_1 = \phi_2 - 90^\circ = S \tag{8}$$

Assuming symmetry, the average of the two times of eq.5 would be exactly the sought quantity: the time t_{SS} of the S.Solst.

G3 What very slightly but aggravatingly upsets the ideal eq.5 situation is the Earth's elliptical orbit. The vital elements were in -157 and thereabouts:

$$\text{Apogee } A = 66^\circ.1 \qquad \text{eccentricity } e = 0.0176 \tag{9}$$

The non-uniform solar motion entailed by the asymmetry[7] of the Sun's elliptic motion causes a *systematic error* that becomes quadratically **larger** (eq.13), the larger the number of days d on either side of the Solstice one chooses to take observations at — even while the process' *random* error becomes **smaller** for greater d (eq.18). So picking the ideal d is a delicate choice (§J), whose pitfalls we now examine. [Note: Many equations to follow here are approximations — though marked as equalities if the roughness is slight.]

H Charybdis

H1 An equation for the asymmetry-caused longitudinal systematic error q, of an Equal-Altitudes-obtained t_{SS}, may not have been previously published; so we have derived (and have substituted eq.9 values into) the following simple formula for q as a function of S, the number of longitude-degrees on either side of the solstice one chooses to start & finish at:

$$q = \frac{\pi e \cos A}{3} S^2 = 0'.0075 S^2 \tag{10}$$

with q in arc-minutes and (again) S in degrees.

H2 We all know the Sun moves about 1°/day, so obviously S is nearly equal to d — near enough for the difference to be largely ignorable here. Nonetheless, we supply useful approximations, relating q to the asymmetry-caused solstice-error H in hours, for solar motion near a Summer Solstice during Hipparchos' era,

$$H = \frac{24 \cdot 365^{\mathrm{d}}.2425}{60 \cdot 360^\circ} q/(1 - 2e\sin A) = 0.42q \tag{11}$$

and relating S to d:

$$d = \frac{365^{\mathrm{d}}.2425}{360^\circ} S/(1 - 2e\sin A) = 1.05S \tag{12}$$

H3 Combining eqs.10-12 yields our ultimate desired simple practical formula (valid for the range of d that knowledgeable ancients would wish to use) expressing systematic error in hours H as a function of the Equal-Altitudes symmetric (ere&aft) interval d in days:

$$H = \frac{-0.0075 \cdot 0.42}{1.05^2} d^2 = -0.0029 d^2 \tag{13}$$

where the minus-sign reflects that for -157 the error of the Equal-Altitudes Method will cause naïvely (eq.5) deduced t_{SS} to be too-early by H hours.

[7] Things were easy in 1245AD, when the solar apogee arrived at longitude 90°. Had this obtained in Hipparchos' era, our entire discussion of asymmetry here would be superfluous.

F Expanding Pair-Analysis

But we go beyond Rawlins 1991W §N14 by computing e (or r) for all 6 pair, thus adding 4 new data (not in *ibid*) that ultimately reveal (§§G1-G3) the cause of Hipparchos' fake.

F1 The three Trio A results for e, recalling R of eq.5 (& including 60^{p}-based values):

$$\begin{aligned} e_{\text{A2-A1}}/R_{\text{A}} &= 334'18''/3144' = 6^{\mathrm{p}}23' \\ e_{\text{A3-A2}}/R_{\text{A}} &= 529'06''/3144' = 10^{\mathrm{p}}06' \\ e_{\text{A1-A3}}/R_{\text{A}} &= 272'04''/3144' = 5^{\mathrm{p}}12' \end{aligned}$$

F2 The three Trio B results for r, recalling R of eq.6:

$$\begin{aligned} r_{\text{B2-B1}}/R_{\text{B}} &= 247'30''/3122'30'' = 4^{\mathrm{p}}45' \quad \Longleftarrow \\ r_{\text{B3-B2}}/R_{\text{B}} &= 248'35''/3122'30'' = 4^{\mathrm{p}}47' \\ r_{\text{B1-B3}}/R_{\text{B}} &= 261'54''/3122'30'' = 5^{\mathrm{p}}02' \end{aligned}$$

F3 For Trio B, it looked to Hipparchos like there was sufficient consistency to justify taking any of the values as a good approximation to r. (The consistency was at best luck,[12] given the poorness of the EH orbit producing input ϕ data.) He chose the top line (arrowed B2-B1) to publish as his r solution. It agrees perfectly with his attested Trio B r (eq.4).

F4 [Added 2012 September. Dennis Duke notes: for pre-fixed $g_\circ$&$\epsilon_\circ$, e (or r) is findable from 1 eclipse. Equation: $-1/e = \cos g + \sin g/\tan(\phi - f)$; yet results don't recover attested e&r (§C1), while §§F2&G2 fit. But why did Hipparchos use the harder Pair Method? [a] He (& Rawlins 1991W §N12) knew it finds e (or r), pre-fixing only $g_\circ$ but not yet $\epsilon_\circ = 178^\circ$ [fn 9; eq.8]. (***NB***: Ptolemy's precise-to-arcmin Ant 1 [137.547] solar&lunar $\epsilon_\circ$ descend from Phil 1 [−322.148] round $\epsilon_\circ$ [Rawlins 1991W eq.8].) [b] Results for pairs tend to be more consistent than for single eclipses. [c] A perigee eclipse (A3) isn't useless when paired. [d] Weather blocked most trios' full capture (thus Hipparchos' resort to 200BC & 382BC data?), so pair-analysis pre-fixing $g_\circ$ (or $\epsilon_\circ$) may have been common.]

G Hipparchos Fakes the Impossible for −381/12/12: A 179° True-Elongation Lunar Mid-Eclipse!

G1 But Hipparchos saw that Trio A's results (§F1) for lunar e were dreadfully inconsistent. (Mostly due to Babylonian observational time-errors [Rawlins 1991W fn 223] & to Frankensteinorbit's solar e being more than 50% higher than reality.) So he[13] committed a crime against science: finding that altering ϕ_{A3} by -1° dramatically "saved" the situation (fn 11), he made the alteration, a sleight that changed the Trio A input data (from §E4) to:

$$\begin{aligned} \delta_{\text{A2-A1}} &= 172^\circ 53' - 179^\circ 46' 07'' = -6^\circ 53' 07'' \\ \delta_{\text{A3-A2}} &= 175^\circ 07' - 173^\circ 08' 04'' = +1^\circ 58' 56'' \\ \delta_{\text{A1-A3}} &= 12^\circ 00' - 7^\circ 05' 49'' = +4^\circ 54' 11'' \end{aligned}$$

G2 Which converted the results of §F1 instead to:

$$\begin{aligned} e_{\text{A2-A1}}/R_{\text{A}} &= 334'18''/3144' = 6^{\mathrm{p}}23' \\ e_{\text{A3-A2}}/R_{\text{A}} &= 327'39''/3144' = 6^{\mathrm{p}}15' \quad \Longleftarrow \\ e_{\text{A1-A3}}/R_{\text{A}} &= 336'33''/3144' = 6^{\mathrm{p}}25' \end{aligned}$$

[12] The seeming good luck of Trio B's consistency was bad luck, since it deluded Hipparchos into expecting similar consistency for Trio A; so when it didn't happen, he made it happen: §§F3-G1.

[13] Eq.6's miscue indicates that at least the R of Trio A & Trio B were computed by distinct members of a hitherto (fn 6) hypothetical Hipparchan stable. Note obvious parallel to the problems producing the few faked stars (Rawlins 1992T & Rawlins 1993D) of Tycho Brahe. (Known to have had a stable.)

motion) required tables for usage, ***which would take time to compile***, thus Hipparchos' continuing use of the old EH tables for a little while after establishing the PH orbit. This situation is consistent (§E2) with the −382-381 longitudes (for the early eclipse catalog, long before the analyses of trios) being computed later than the −200-199 ones — presumably c.−145, the time when (‡2 §O3) he established the iconic (‡2 §O1) PH orbit.

E4 Continuing in Hipparchos' footsteps, we now compute (eq.7) the δ for all six eclipse-pairs. (Notice that for each trio, the sum of the three δ is zero.)

$$\begin{aligned}
\delta_{\text{A2-A1}} &= 172°53' - 179°46'07'' = -6°53'07'' \\
\delta_{\text{A3-A2}} &= 176°07' - 173°08'04'' = +2°58'56'' \\
\delta_{\text{A1-A3}} &= \ \ 11°00' - \ \ \ 7°05'49'' = +3°54'11'' \\
\delta_{\text{B2-B1}} &= 180°20' - 188°41'24'' = -8°21'24'' \\
\delta_{\text{B3-B2}} &= 168°33' - 159°46'31'' = +8°46'29'' \\
\delta_{\text{B1-B3}} &= \ \ 11°07' - \ \ 11°32'05'' = -0°25'05''
\end{aligned}$$

E5 Via likely roundings, Rawlins 1991W (§§M9&L2) reconstructs Hipparchos' absolute times of the six eclipses. Hipparchos' mean anomaly for each eclipse was found through multiplying his traditional Aristarchan lunar mean anomalistic motion (*ibid* eqs.6-7: not quite the same as *Almajest* 4.7's) by the time since his theories' epoch (Phil 1 = −323/11/12: ‡2 §O3), and then adding the result to his equally Aristarchan lunar mean-anomaly-at-epoch $g_\circ = 82°$ (Rawlins 1991W eq.9),[9] which relates to apogee-at-epoch $A_\circ$ thusly:

$$g_\circ = \epsilon_\circ - A_\circ = 178° - 96° = 82° \tag{8}$$

E6 This produces the following mean anomalies:

$$\begin{aligned}
g_{\text{A1}} &= 224°1/3 & g_{\text{B1}} &= 297° \\
g_{\text{A2}} &= \ \ 24°1/3 & g_{\text{B2}} &= 105°5/6 \\
g_{\text{A3}} &= 177°44' & g_{\text{B3}} &= 246°
\end{aligned}$$

E7 Ere 1991, all presumed (from *Alm* 4.5) Hipparchos used eclipse-trios to find 3 unknowns simultaneously: e, $g_\circ$, & $\epsilon_\circ$. Rawlins 1991W §§ N5f found he used not trios (which produce no matches: §A2) but eclipse-*pairs* (which do: §§F2&G2), thereby seeking only e, while appropriating $g_\circ$ (see Rawlins 1991W §N10!!) and ultimately it seems (§F4) $\epsilon_\circ$ from a prior astronomer: eq.8. [A paper (§K1) rejecting the Pair Method cites *Alm* 4.5's belief that Hipparchos' method is that of *Alm* 4.6&11, meanwhile accepting the paper's own 3438-base alteration of same.] To find e from an eclipse-pair, trigmaster Hipparchos used the pure-trig Pair Method (easier&clearer than Ptolemy's eclipse-trio Simultaneous Method [§I1], though inferior in result):[10] for any eclipse-pair we specify their 2 mean anomalies g (already computed at §E6) as α & β and use them with δ (§E4) in the following 3-step trig procedure (perhaps unknown during the 21 centuries up to Rawlins 1991W §N13):[11]

$$U = -[(\cos\alpha + \cos\beta) + \cot\delta(\sin\alpha - \sin\beta)]/2 \tag{9}$$

$$V = \cos(\alpha - \beta) + \cot\delta\sin(\alpha - \beta) \tag{10}$$

$$e \text{ or } r = R/(U + \sqrt{U^2 - V}) \tag{11}$$

[9] Where $\epsilon_\circ = 178°$ is Aristarchos' (later Hipparchos'&Ptolemy's) lunar mean-longitude-at-epoch; $g_\circ = 82°$, his mean-anomaly-at-epoch; $A_\circ = 96°$, his apogee-at-epoch (epoch = Phil 1: §E5).

[10] The poorness of Hipparchos' results alone suggests a primitivity incongruent with the sophisticated Simultaneous Method. (And the inconsistent consistencies of the 60^{p}-based values of §F2 vs §G2 suggest worse.) As earlier realized by van der Waerden and shown at Rawlins 1991W (§S1) & Rawlins 2009E, Hipparchos wasn't an outstandingly able math-theoretician, though (contra Duke at §§K1&K4 & Jones at Rawlins 2009S §§G2-G3) *an unerringly reliable computer*: here & Rawlins 2009S Fig.1.

[11] This presumes that Hipparchos didn't solve pairs by trial. Note: all §E6's g are round fractions (suggesting that some eclipse-data might've been slightly adjusted), except for the near-perigee (thus *very* sensitive) A3 case, where 1-unknown math (via §F4's equation) upon pre-doctored A3 yields e both outsized & *negative*. (An alternate explanation for Hipparchos' fudging eclipse A3 by −1°.)

I Scylla

We now turn from systematic error to *random error*.

I1 If solar altitude h could be measured perfectly, the foregoing Charybdis section would be a complete error analysis. But the measure of S is from visual determination of altitude h, which can be measured to no better than 1/10000 of a radian (Rawlins 2002B eq.1), called here the Optimal standard-deviation σ_{Opt} for human vision — and contrasted with widely-assumed Ordinary visual discrimination (oft apt in-practice: §B4), $\sigma_{\text{Ord}} \doteq 1'$:

$$\text{Optimal Discrim } \sigma_{\text{Opt}} \doteq 1/10000 \text{ radian} \doteq 1'/3 \qquad \text{Ordinary Discrim } \sigma_{\text{Ord}} \doteq 1' \tag{14}$$

Eq.14 causes an uncertainty (σ_{SS}) in an Equal-Altitudes-determined S.Solst time t_{SS} which requires statistical evaluation. So, to find an accurate t_{SS}, we initially need to know how strongly h-uncertainty produces uncertainty in hours of solar motion.

I2 Since LAN solar h and solar declination δ virtually differ by a constant, we start by gauging the statistical relation of longitude ϕ's uncertainty σ_ϕ to h's uncertainty σ_{h}:

$$\frac{\sigma_{\text{h}}}{\sigma_\phi} = \frac{\Delta h}{\Delta\phi} \doteq \frac{\Delta\delta}{\Delta\phi} = \tan\epsilon\sin S \doteq \frac{\pi\cdot\tan\epsilon}{180}S \tag{15}$$

(σ_ϕ & σ_{h} in arcmin), where obliquity ϵ was 23°.7 in Hipparchos' era. Also (a statistical parallel to eq.11), we find the effect of σ_ϕ upon observed t_{SS}'s uncertainty σ_{SS} (in hours):

$$\frac{\sigma_{\text{SS}}}{\sigma_\phi} = 0.42/\sqrt{2} \tag{16}$$

where the $\sqrt{2}$ reflects t_{SS}'s dependence (eq.5) upon not one but two h measures, averaged.

I3 Combining eqs.15, 16, & 12 establishes standard-deviation ratios:

$$\sigma_{\text{SS}} = \frac{0.42}{\sqrt{2}\tan\epsilon\sin S}\sigma_{\text{h}} \doteq \frac{180\cdot 0.42}{S\pi\sqrt{2}\tan\epsilon}\sigma_{\text{h}} \doteq \frac{39}{S}\sigma_{\text{h}} = \frac{41}{d}\sigma_{\text{h}} \tag{17}$$

We note that when $S = 0$ (the Swerdlow-Moment: §B2), uncertainty (σ_{SS}) in an Equal-Altitude-Method-obtained S.Solst-time is infinite — as it obviously should be.

I4 To evaluate t_{SS}'s uncertainty σ_{SS} as a function of d for Optimal and Ordinary visual discrimination, we exploit eq.17 by substituting into it eq.14's respective values for σ_{h}:

$$\text{Optimal } \sigma_{\text{Opt SS}} \doteq \frac{14}{d} \qquad \text{Ordinary } \sigma_{\text{Ord SS}} \doteq \frac{41}{d} \tag{18}$$

I5 The above considerations show that accuracy to well within the ancient-cited (*Almajest* 3.1) allowance of 6^{h} error was possible, so it should be no surprise that all three of the firm outdoor solstices of Table 3 are accurate within the uncertainty-estimates of the present section: after all, for accurate data correctly rounded to 6^{h} precision, the implicit error-range is $\pm 3^{\text{h}}$.

J Balance

J1 We next weigh the tricky choice an ancient solstice-observer had to face. If chosen d is too small, he is prey to the quirky Scylla of corruption of his project by random error of indeterminate size and even sign. But if the ancient astronomer over-counters that danger by opting for too-large d, he leans too near Charybdis' tranverse swirl and thus intolerable systematic negative error. (Hartner 1977 & Thurston 2001 cite pre-telescopic observers' d ranging from 45^{d} [eq.13: $-1^{\text{d}}/4$ syst.error] to 8^{d} [$1^{\text{d}}/4$ random error for Ordinary eq.18, 2^{h} for Optimal].) To estimate an ideal Balanced interval d_{BAL}, we can combine eqs.13&18 to ensure that H and σ_{SS} are about the same size:

$$\text{Optimal } d_{\text{BAL}} = \sqrt[3]{14/0.0029} \doteq 17^{\text{d}} \qquad \text{Ordinary } d_{\text{BAL}} = \sqrt[3]{41/0.0029} \doteq 24^{\text{d}} \tag{19}$$

But we must not forget that: [1] The two errors (eqs.13&18) are of quite different type. [2] An ancient scientist would instinctively sense eq.18. [3] There's no evidence that any ancient (or modern?) knew of eq.13; if he had, he'd have compensated, either by correcting for it (thus positive errors in Table 3?) or suppressing its effect via modest-sized d.

J2 Eq.19 indicates that 20^{d} is about the best choice for d. Substituting into eq.18:

$$\text{Optimal } \sigma_{\mathrm{SS}} \doteq 0^{\mathrm{h}}.7 \qquad \text{Ordinary } \sigma_{\mathrm{SS}} \doteq 2^{\mathrm{h}} \tag{20}$$

Neither creates a problem. And eq.13 gives for $d = 20^{\mathrm{d}}$ a systematic error:

$$H \doteq -1^{\mathrm{h}}1/6 \tag{21}$$

Virtually negligible, and (since we listened to eq.19) roughly equal to eq.20's random errors Indicated net accuracy by combined effect of eqs.13&18 is easily within, indeed, a good deal better than, the $1^{\mathrm{d}}/4$ outer error-possibility (§I5) Hipparchos cites. This grants assurance the 3 firm solstices of Table 3 are validly non-accidental in their nearness to reality.

J3 Kallippos' non-trivial positive error ($+3^{\mathrm{h}}$) has the wrong sign (§H3) for serious systematic error, so he may have used $d \doteq 10^{\mathrm{d}}$ when measuring the -329 S.Solst, leaving him open to ordmag 1^{h} of random error. But it is hard to tell, since $1^{\mathrm{d}}/4$ rounding obscured the exact hour measured. It was natural for Kallippos to round thusly since Greek calendars always started on a quarter-day mark. This particular observation was superlatively calendaric, in that the S.Solst occurred closer to the New Moon (within ordmag 1^{h}) than any S.Solst for ordmag a century, which is presumably why Kallippos chose this moment to launch his famous *luni-solar* calendar. Also, the most recent eclipse (exceptionally cited by Pliny 2.72 & *GD* 1.4.2) visible to Kallippos was the -330/9/20 Gaugamela (Arbela) eclipse, the fame of which he — as Alexander's astronomer — may have enhanced.

J4 The time-interval, between Meton's truncated (Table 1) -431 dusk S.Solst and Kallippos' dawn S.Solstice turned out to be exactly divisible by $365^{\mathrm{d}}1/4$ (eq.1). Thus, Kallippos' calendar — evidently due to a truncation that caused a huge -17^{h} error (Table 1) in the earlier datum! — became the earliest known to have used the $365^{\mathrm{d}}1/4$ year (§C1). And, again, Kallippos was the 1st astronomical calendarist to (crucially) start a still-extant calendar without rounding his contemporary founding-S.Solst to conventional day-epoch (which we know he didn't do, because the day interval from Meton's day-epoch isn't integral: §C1), properly starting it instead at what he estimated to be the nearest quarter-day point, 6^{h} in the morn, his decision reinforced by that hour's proximity to an extremely rare close-confluence (§J3) of S.Solstice & New Moon (the latter nearer dawn than midnight). [Greeks defining New Moon by true-longitudinal syzygy, rather than following Babylon's crude First-Visibility definition, indicates who was ahead in math astronomy by 330 BC.]

J5 The error of Hipparchos' -134 S.Solst (eq.4) is trivially negative (Table 3), as it should be; though, again, $1^{\mathrm{d}}/4$ rounding muddies our evaluation. Regardless, we can say that Hipparchos achieved the most[8] accurate of all surviving ancient solstices. (Unless we count the phantom solstice of -157/6/26: §M4 & Table 2.)

K Hipparchos' −157/6/28 Dawn Summer Solstice

[Thanks to *DIO* refereeing, albeit (uncharacteristically) late in this case, §§K-P have been rethought, recalculated, & rewritten (2018 Winter): prior mistakes fixed & new finds added.]

K1 Hipparchos c.-157 was using past records of eclipse-times to start building his famous 600^{y} eclipse canon (§M2; Rawlins 1991W §M7), a list which included Hipparchos-computed solar longitudes ϕ for each eclipse's historically known time. Later, these ϕ were brought in when he analysed eclipse-trios. In Rawlins 1991W §K9, we found that his math-analysis of eclipse-Trio B (‡3 fn 5) used ϕ computed (for each eclipse-time) from what we dubbed his "EH Orbit" (founded -157), which was afflicted with terrible apogee $A = 44°$ & eccentricity $e = 3^{\mathrm{p}}1/4$, by taking (Rawlins 1991W §K8) EH's S.Solst — via indoor math — from Kallippos' -329/6/28-epoch calendar (accumulated error $+1^{\mathrm{d}}.3$ in the 172^{y} interim), due to its over-long year-length, Y_{K} (§C1). From §C1 & eq.1 (see Tables 2&3):

$$\text{EH Summer Solst} = -329/6/28\ 1/4 + 172 \cdot 365^{\mathrm{d}}1/4 = -157/6/28\ 1/4 \tag{22}$$

[8] Again: this may merely be due to the actual -134 S.Solst being accidentally closer to a quarter-day mark than those of -329 & -146. See fn 22.

D Ordmag's Debut: Distance to the Sun

In the 2012 June *Astronomy* (p.31), Bill Andrews asks: when historically did the idea of using order-of-magnitude (ordmag) arise? The answer resides in our eq.5: Greeks pioneered adoption of order-of-magnitude, naturally resorting to powers of ten to gauge the too-uncertain-for-precision solar distance in Earth-radii. Remote from their subjects, astronomers were the inevitable inventors of ordmag. Sun-distance estimates by eminent ancient scientists follow (superscript e = Earth-radii). Eratosthenes: 100^{e} (Rawlins 2008Q eq.12); early Aristarchos & mid-career Hipparchos: 1000^{e} (Rawlins 2008R §D1; above eq.5); late Aristarchos, Archimedes, & Poseidonios: 10000^{e} (Rawlins 2008R eqs.14-15).

E Exact Origins of Hipparchos' Eccentricity e & Epicycle-Radius r

E1 For each eclipse-pair, *Almajest* 4.11 provides two pair of data (already listed here at §B2): time-interval Δt in days and true longitude-interval $\Delta\phi$ in degrees; Δt is multiplied times mean lunar motion — namely, Kallippic solar motion (Rawlins 1991W §§K9&M4) plus long-canonical Aristarchan (Rawlins 2002A) synodic lunar motion (‡1 eqs.4&5) — which yields Δf, the mean lunar motion in degrees during the interval between the eclipse-pair. For any pair, non-uniform motion causes a gap between mean-longitude-difference Δf and the true-longitude difference $\Delta\phi$ — where, again: each $\Delta\phi$ is already supplied explicitly (§B2) at *Almajest* 4.11. (Though, Hipparchos had illegitimately [§G4] altered A3's longitude by $-1°$, as will be shown below: §G1.) This gap is labelled δ:

$$\delta = \Delta\phi - \Delta f \tag{7}$$

True longitude ϕ was computed in each case by Hipparchos from the solar orbit he had adopted at the time of the calculation. (These orbits' poorness created fateful errors in the ϕ values, which were to undo [§G1] the very Pair Method [§E7] Hipparchos used.)

E2 The longitudes ϕ in Hipparchos' famous catalog of 600^{y} of eclipses were presumably computed well before his trio-calculations. Said catalog was naturally compiled going backward in time, a judgement which becomes more than guesswork when we find that the -200-199 true longitudes ϕ were calculated from an inferior early "EH" solar orbit[8] whose 4 elements are given at Rawlins 1991W §K9 (along with its empirical bases: *ibid* §§K4-K9), while the parallel -382-381 longitudes were found from a solar orbit that constituted a relic of the later transition from EH to his famous "PH" orbit (preserved at *Almajest* 3.1-7), which is why it bore elements from both EH & PH: a stitched-together element-mix quasi-facetiously called "Frankensteinorbit". (See Rawlins 1991W § M4-M5.)

E3 Determining chronological order (*ibid* §M5): the 2 elements of Frankensteinorbit drawn from PH (solar apogee-at-epoch & mean-longitude-at-epoch) were constants — thus available for immediate use — while the 2 elements drawn from EH (eccentricity & mean

[8] Duke 2008W, *JHA*'s August Pb paper, rejects (‡2 fn 10) all of DR's 3 Hipparchos orbits (EH, Frankenstein, & [Rawlins 1991H] UH), deeming them "neither conclusive nor satisfying" since (emph added) "parameters deduced from trio analyses are very sensitive to small *changes* in the input data" (shouldn't that read "small *errors*"? — see ‡2 fn 10 items [4]-[5]), from Duke 2008W's unique delusion (‡1 §B4) that Greek solar data averaged $15'$ error. Only citation relating to target DR is nonexplicit: a *JHA*-doctored note; see *DIO 6* §§D1&H2 & fn 20. (*JHA* refereeing. Again.) But uncited Rawlins 1991W fn 205 explored this sensitivity, *thus DR didn't just compute orbit-elements from trio* ϕ but the reverse: EH&UH were instead initially founded upon Hipparchan cardinal-point data (firm or reasonably reconstructed: ‡2 §§K or Rawlins 1991W §K), then *tested against* extant trios' ϕ. Further testing found that a meld (*correctly ordered, chronologically*: §E3) of EH&PH fit Trio A's ϕ, thereby establishing Frankensteinorbit & dating it to the -145 [V.Equinox] (‡2 §O3). Doubting UH requires rejecting ‡2 eqs.3-4. (Contra pp.23-24 of the very Jones 2005 paper cited by Duke 2008W p.289 n.9.) UH — incl. above ‡2 eq.4 — solved *five* mysteries simultaneously (Rawlins 1991H): [a] why Aristarchos & Hipparchos solstices are (‡2 §C1) sole hourless *Almajest* 3.1 Sun data; [b] all 3 Trio C ϕ (*Almajest* 5.3&5); [c] $5'$-PH-discrepant f of Trio C's 2nd ϕ; [d] $0°.2$ amplitude of AncStarCat zodiac stars' periodic error; [e] Moon-phase when AncStarCat fundamental stars observed. (Also, suggestive: Hipparchos' UH&AncStarCat -127 A.Eqx epoch follows Meton's S.Solst by $304^{\mathrm{y}}1/4$, ***exactly*** 1/16th [Rawlins 2002A fn 17] of Hipparchos' 4868^{y} Great Year: ‡2 §P4.)

B2 The Trio A time-intervals and longitude-intervals (*Almajest* 4.11):

$$A2 - A1 : 177^{\mathrm{d}}13^{\mathrm{h}}3/4 \;\; 173^\circ - 1^\circ/8 \qquad A3 - A2 : 177^{\mathrm{d}}01^{\mathrm{h}}2/3 \;\; 175^\circ + 1^\circ/8 \quad (1)$$

The Trio B time-intervals and longitude-intervals (*idem*):

$$B2 - B1 : 178^{\mathrm{d}}06^{\mathrm{h}} \;\; 180^\circ 20' \qquad B3 - B2 : 176^{\mathrm{d}}01^{\mathrm{h}}1/3 \;\; 168^\circ 33' \quad (2)$$

C Precisely Solving the Origin of Hipparchos' Lunar Distances

C1 *Almajest* 4.11 supplies Hipparchos' disparate results for lunar distance R and eccentricity e (or, equivalently,[6] epicycle-radius r):

$$\text{Trio A}: \qquad R = 3144 \qquad e = 327\ 2/3 \quad (3)$$

$$\text{Trio B}: \qquad R = 3122\ 1/2 \qquad r = 247\ 1/2 \quad (4)$$

For decades after Toomer 1973, two of these desiderata, the lunar-distances R (3144 & 3122 1/2) were held by the political center (traditionally overlapping the O.Neugebauer klan) to be by-products of elaborate calculations, *during* which the prime sought-numbers *occur* and are then frozen-in-midstream: i.e., *the very yardstick-radius of the lunar orbit* (R) is supposed to just fall out of the process on-the-fly (§§I8-I9 & §§I12-I13) — unheard-of in Greek (or any other) mathematical astronomy. Including Indian.

C2 Rawlins 1991W precisely reproduced the R values by showing both were instead *fixed at the outset*, finding 3144 by just applying simple trig to Aristarchan data (Hipparchos was partial to such: *ibid* fn 243 & §O8): half-Moon elongation 87° (or 3° from quadrature: ‡1 §C2), and Sun at distance 1000 Earth-radii (§D). From *ibid* eq.23, we have:

$$R(\text{Trio A}) = 1000^{\mathrm{e}} \cot 87^\circ = 1000^{\mathrm{e}} \tan 3^\circ \doteq 52^{\mathrm{e}}24'28'' \doteq 3144' \quad (5)$$

Perfect match to the attested value for Trio A's R (fn 1). Notice that this R has thus established a startling revelation: measuring the lunar distance in solar-based units is the mark of a heliocentrist. And recall that all input data in eq.5 came from Aristarchos, who was the public pioneer of heliocentrism. (See also fn 6.)

C3 True, eq.5 doesn't yield Trio B's $R = 3122\ 1/2$. But instead of an impediment, this discord is about to be revealed as the clincher for eq.5's source.

C4 One of the commonest ancient and modern misreadings of Greek numbers is the confusion[7] of sixtieths with fractions. Evidently Trio B's computer misread Trio A's $52^{\mathrm{e}}24'$ (eq.5) as $52^{\mathrm{e}}1/24$, thus (as 1st discerned at Rawlins 1991W eq.24)

$$R(\text{Trio B}) = 52^{\mathrm{e}}1/24 = 52^{\mathrm{e}}02'1/2 = 3122'1/2 \quad (6)$$

— the precise attested value for Trio B's R (eq.4; fn 1). This delightful confirmation of eq.5's heliocentrist revelation boosts our certitude that heliocentrism — lethally suppressed though it was (Rawlins 1991P) — carried on quietly in the astronomical community (as it did in 18th century France), even showing up in the work of geocentrist Hipparchos.

No absolute Hipparchan value of any's hour or longitude survive explicitly. (Strictly differences: *Almajest* 4.11.) But all of these dozen absolute data were precisely reconstructed at Rawlins 1991W §§M9-10 & L2-3.

[6] Is it indicative that Hipparchos started with the eccentric lunar theory, rather parallel to the heliocentrists' model for planets, but later moved over to the epicyclic lunar theory, parallel to the geocentrists' model? Note it was the earlier (Trio A) computer who introduced (§C2) heliocentrist measure into determining R.

[7] For examples, see Rawlins 1991W §O3. Also Rawlins 1994L ‡3 fn 39.

K2 Relative to the present analysis, the key point to notice is this: *Hipparchos in −157 would not have computed a solstice from a predecessor's calendar unless he didn't yet know how to observe a solstice reliably*. (The poetic irony here is that before his career was done, Hipparchos left us [§J5] THE most accurate outdoor-observed solstice that survives from antiquity, the error in which is merely about an hour. See Table 3.)

K3 But if the young Hipparchos needed to resort to an earlier astronomer's calendar to obtain his −157/6/28 dawn solstice (eq.22, used for constructing his EH solar orbit of −157: §K1 or Rawlins 1991W §§K8-K9), then where did his newly discovered (§A) −157/6/26 solstice come from? And when? Rigid impediment to casual thinking hereabouts: his calculational use of the EH orbit's tables as late as −145 (Rawlins 1991W §§M4-M6) shows that the 6/26 replacement-improvement *was not adopted immediately*.

K4 Duke's idea (people.sc.fsu.edu/~dduke/Duke-Neugebauer-2.pdf) that the −157/6/26 solstice was empirically determined to have occurred at 21^{h}, seems to be based upon his perceptive recognition of Meton's fingerprint: multiplying the papyrus' tropical yearlength (convincingly extracted from it by Tihon 2010 p.5)

$$Y_{\mathrm{p}} = 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/309 = 365^{\mathrm{d}}.24676 \quad (23)$$

times the 274^{y} gap since Meton, and adding the product to Meton's S.Solst, as misunderstood by Hipparchos & Ptolemy (§C1), produces:

$$\text{S.Solst} = -431/6/27\ 1/4 + 274 \cdot Y_{\mathrm{p}} = -157/6/26\ 20^{\mathrm{h}}43^{\mathrm{m}} \doteq -157/6/26\ 21^{\mathrm{h}} \quad (24)$$

However: [a] All known Hipparchos cardinal point data are rounded to the quarter-day. [b] In reverse, eq.24's 21^{h} time produces yearlength about $365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/313$, not $1^{\mathrm{d}}/309$, and so doesn't solve eq.23's origin. [c] In −157, Hipparchos wasn't yet (§L4) sky-observing at a level likely to find an accurate solstice such as that proposed. [d] The papyrus says that the −157/6/26 solstice occurred at an unknown number of hours of the *day* not night.

K5 Potential resolution of [a]-[d]: if the papyrus said "12 hours of the day" (18^{h} or 6 PM), that would make the gap from Meton (−431/6/27 1/4) to Hipparchos (−157/6/26 3/4) equal to $100077^{\mathrm{d}}1/2$. But the ancient scholar who created eq.23 could have accounted for seasonal hours' solstitial day-lengthening, taking $14^{\mathrm{h}}3/4$ as the nearest klima (of *Almajest* 2.6's traditions) to a mean between Athens' & Nicaea's *GD* Book 8 longest days (Diller 1984):

$$Y_{\mathrm{s}} = (100077^{\mathrm{d}} + 14^{\mathrm{h}}3/4)/274 \doteq 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/309 \quad (25)$$

(We here assume early Hipparchos didn't know of or ignored small longitude differences.) Had the $14^{\mathrm{h}}5/8$ Athens klima (*GD* 3.15.22) been used, the remainder would've been $-1^{\mathrm{d}}/308^{\mathrm{y}}$, perhaps an alternate value, as suggested by the *P.Fouad* 267A left column's remainder $+3^{\mathrm{d}}/308^{\mathrm{y}}$ (Tihon 2010 p.7). Either way *Fouad* Hipparchan precession appears (but note fn 16) exactly or nearly $4^{\mathrm{d}}/308^{\mathrm{y}} = 1^{\mathrm{d}}/77^{\mathrm{y}} \doteq$ (*ibid* pp.6-7) $1^\circ/78^{\mathrm{y}}$ (vs actual $1^\circ/72^{\mathrm{y}}$ then), hinted at *Almajest* 7.2 ("not less") but not explicitly relayed there (& a better figure than Ptolemy's $1^\circ/100^{\mathrm{y}}$). [Or reverse? Prior $1^{\mathrm{d}}/77^{\mathrm{y}}$ precession estimate times 4→308^{y}?]

L When Was the −157/6/26 3/4 Solstice Observed?

L1 The seemingly odd title of this section is not meant facetiously. (Though it puts one in mind of humor at the level of what-was-the-color-of-George-Washington's-white-horse?) It is deliberate — because we are faced with a weird contradiction, *two different dates for the same event*, the −157 S.Solst: −157/6/28 1/4 (§L2) vs −157/6/26 3/4 (papyrus: §K5).

L2 Rawlins 1991W[9] (see §K1 above) has shown that Hipparchos' eclipse-trios A&B cannot closely enough fit *Almajest* 4.11's intervals for a solar eccentricity less than 3^{p}. (And

[9] Parts of Rawlins 1991W are written in an anti-tyrannical spirit which is bound to offend anyone unfamiliar with the cult that has for decades financially puppetized most of the history of ancient astronomy community, to its tragic cost in competence, refereeing, neutrality, and most importantly: valid history. If the History of Science Society can (fn 10) wince and stomach DR's idiosyncratic writing style [once upon a time! — see Rawlins 2018A], *in order to get at the truth* of Hipparchos' early observations & lunisolar elements, then fair-minded individual investigators ought to be able to manage same — for *P.Fouad* 267A will never be understood if §§K-O are discounted.

Trio B can't fit an apogee A above 50°. Both limits are grossly discrepant vs the standard *Almajest* PH orbit's $e = 2^{\mathrm{p}}1/2$, $A = 65°1/2$.) Rawlins 1991W found that this clash is neatly accounted-for by a huge error (over 1^{d}!) in solstice — and that the EH orbit satisfying[10] this glaring oddity is also consistent (like no later A.Eqx) with the quite erroneous −157 Autumn Equinox (off by 11^{h}, nearly half a day!) reported at *Almajest* 3.1 and is consistent with a −157 solstice at 6/28 1/4 (§L1; Table 2), exactly[11] where the Kallippic calendar has it. EH was used by Hipparchos until his adoption of the later-canonical PH orbit in −145, when EH's rôle in Trio A's Frankenstein-orbit solution proves Hipparchos anchored at S.Solst −157/6/28 1/4 right up until −145, not at the *Fouad* papyrus' −157/6/26 3/4.

L3 If *Fouad*'s −157/6/26 solst were outdoor-observed, why was its adoption and use (in founding PH) delayed until 12^{y} later? Rawlins 1991W §M reveals a lucky glimpse of Hipparchos' −145 solar math, as he semi-shifted into upcoming full adoption of PH. **NB**: *ibid* §M6 *must be read with care to appreciate the non-random split in Frankenstein orbit* (half its elements are EH; other half, PH) which Hipparchos used for computing Trio A, and ***the obvious explanation of this striking bifurcation***: he of course temporarily retained the tabular EH elements, since the 2 necessary PH tables would take awhile to create (eventual results: *Almajest* 3.2&6), while swiftly adopting non-tabular PH elements (constants A&ϵ_{o}), as carefully explained at Rawlins *loc cit*: when computing eclipse Trio A in −145 early Spring (§N5), Hipparchos ***was in mid-transition from EH to PH***.

L4 *Ibid* §§K2-3 noted several symptoms of roughness in Hipparchos' work from −161 to −157, and wondered if earlier he was yet even using vertical instruments, before[12] he by

[10] Having *once* (*DIO 11.2* cover) been righter than *one* part of *one* (*non-DIO*) DR paper, bloodtasting Jones&Duke have for 10^{y} been on a knowledge-subtractive mission to trash (in unrefereed forums) as many *DIO* discoveries as possible by ***altering or condemning each's data-base*** (i.e., attacking Strabo, *Almajest*, etc, since DR's invulnerable math keeps leaving no other choice for fanatics kill-bent on denigration) *while for all 6 cases noncitíng the very (reffed) DIO papers targetted*. Hideous details at Rawlins 2018A §§C-G: ***DO NOT MISS***. Similar 7^{th} case: Rawlins 2009S fnn 54-55. Excepting one klima (*ibid* eq.3), the 6 data-sets DR used were standard. ***Until DR solved them***. [1] Duke 2005T pp.170f nakedly (‡3 §§K1&K4) alters Toomer 1973's work to rig matches to *Almajest* 4.11 data which Rawlins 1991W matched tamplessly. [2] The 14 Strabo klimata data, *perfectly-fit honestly* by Diller-DR (Rawlins 2009S Table 2), destroyer Jones 2002E alters by 100 stades but, *uniquely for this flap, displays no table*: such would show his insane (Rawlins 2009S §B6 & fn 55) theories don't fit ***even his own fudged data***. [3] Via cont'd fractions, Rawlins 1999 eqs.3&9 connected Vat. gr. 381's Aristarchan entry to 152^{y}, while [4] Rawlins 2002A eqs.12&13 connected Vat. gr. 191's Aristarchan entry to 4868^{y}, both known Aristarchan intervals. Jones 2010A (p.21 & n.27) reacts by forgery, deleting all accents so no solution is possible (sterility) & non-cites DR, though Jones read *DIO 9.1* on 1999/7/14. [5] Rawlins 1991H & [6] Rawlins 1991W undeniably-accurately recovered 3 Hipparchan orbits' elements from *Almajest* data; so, incredibly, Duke 2008W complains (‡3 fn 8) that, if (like [1] above) he rigs things, the results are too darned sensitive to his proposed *Almajest* re-writes! I.e., he couldn't (§N3) find non-*DIO* elements satisfying the *Almajest* 4.11&5.3&5 data WITH $1^{\mathrm{d}}/4$-rounded times for cardinal-pt ϕ, as valid Hipparchan orbits must, & as the UH, PH, & EH orbits all do. (On the evidence of ‡3 fn 22, Duke rummaged hard to find such.) More sterility. Comments: [a] Don't Duke-Jones know data-tampering is improper? [b] Ever heard of a *single* other case of data-trashing to refute heresy? Much less SEVEN cases, all aimed at the same #1 *JHA*-hate-object? Rawlins 1991W's *double* EH orbit inductions (Trio A & Trio B each lead us to EH) were math-checked & backed by Britton, Thurston 2002S pp.66-67 (Hist.Sci.Soc.) & Curtis Wilson (letter 1994/12/29). One would never know from *JHA*-HAD (JHAD) noncitation that world-class experts encouraged EH. The shocking & unique intercultural implication of eq.4 has long been displayed at the British Museum and accepted by, e.g., Dicks 1994 fn 37, Britton, Thurston 2002S p.62, even Jones 2005 (non-*JHA*) pp.23-24. Outside low-end JHAD, Duke 2008W Table 1 is alone in rejecting eq.4, mis-stating *Almajest* 3.1 contradicts it.

[11] This is not an *ad hoc* adjustment: Rawlins 1985H's novel finding that Kallippos' calendar-founding solstice was at dawn (−329/6/28 1/4) occurred 6^{y} before discovery of the EH orbit using it. Just one more of the many vindications (http://www.dioi.org/vin.htm) of *DIO* findings which keep exasperating our toadily-awesome phalanx of maid-boy *DIO*-assassin-wannabees.

[12] The huge errors in Hipparchos' early Bithynian data (§§L2&L4) could've been from use of ortive amplitudes and-or a mere gnomon (Rawlins 1991W ffn 186&195). Solar altitude errors, if steady,

‡3 Hipparchos' Fake −381/12/12 Mis-Eclipse

His Eclipse Calculations Used Pairs — Not Threesomes
Newly Confirmed By Resolving His One-Degree Fudge
Hipparchan Computations' Mechanical Flawlessness
Greek Invention of Order-of-Magnitude Estimation

A Summary

A1 Hipparchos' work with eclipse-trios (in the 140s BC) was mathematically analysed in 1991 by *DIO*, and all[1] 4 of the lunar-orbit elements (*Almajest* 4.11) Hipparchos had published were precisely elicited thereby at Rawlins 1991W, www.dioi.org/vols/w13.pdf, eqs.19-20&23-24, as generously noted in the History of Science Society's *Isis* during its coverage (Thurston 2002S) of *DIO*'s reconstructions. These 4 solutions transpired through far simpler analysis *and via more ancient-style round-number elements* than prominent prior work (§A2) that failed to reproduce the same 4 data.[2] Analysis' by-product: revelation that Greek science used order-of-magnitude. [DR thanks John Britton & the late Hugh Thurston for thoroughly and expertly verifying all of the mathematical steps of Rawlins 1991W. Also special thanks to Dennis Duke for inspiring, vetting, and tolerating the present paper.]

A2 The 1991 matches are so unanswerably perfect that they have never even been cited by the history-of-astronomy political center, the esteamed "Muffia", which clings to its own goofy old theory (Toomer 1973), though it fits none of the 4 above-cited elements. (Unless one blatantly funnies input data: §K.) Nor does this cult-fave (§§C1, H2, I9&I10) theory explain Hipparchos' 1° data-fudge, a 2000^{y}old puzzle 1^{st} solved here at §G1 by extension of the gratifyingly fruitful 1991 analysis, which also bears a glimmer of early heliocentrism.

A3 Below, we precisely solve (§§C2-G) both trios, achievable because ***Hipparchos' calculations are always mechanically flawless*** (a point helping place his observatory near Lindos: see *DIO 7.1* ‡3 end-Note), our historically key hitherto-implicit finding (Rawlins 1991W, confirmed: Rawlins 2009S Table 2) — ever-denied (e.g., fn 22) by *DIO*-shunners, who can only promote their predictable (Rawlins 1991W §H2 [g]) desperately weird anti-*DIO* pseudo-discoveries by dreaming-up[3] Hipparchos (fn 10) & Strabo (Rawlins 2009S §B6) math errors at will. (Details of 30^{y}-shun's tantrum-origin: see Rawlins 1991W §B.)

B Hipparchos' Data

B1 Our subject here will be two much-discussed ancient lunar eclipse trios: from 383-382 BC (observed in Babylon) and 201-200 BC (observed in Alexandria). The trios are today generally designated as "Trio A" & "Trio B", respectively. (All six dates listed at fn 5.) Both trios were mathematically analysed by Hipparchos c.150-145 BC, during his primitive attempts to improve knowledge of the Moon's nonuniform motion. The empirical data he started with were merely past reports of eclipses' times (& magnitudes & durations), for which he computed true longitudes of the Sun (thus Moon opposite) from his solar tables of the moment.[4] Hipparchos' stated intervals are for two pair from each trio,[5] as follows:

[1] The Hipparchan numbers to be (re)traced here are: 3144, 3122 1/2, 327 2/3, 247 1/2. Rawlins 1991W solved all four to precision given. The paper's calculated reconstructions are reprised below: 3144 (eq.5), 3122 1/2 (eq.6), 327′39″ (arrowed A3-A2 at §G2), 247′30″ (arrowed B2-B1 at §F2).

[2] Rawlins 1991W §P2: all 4 unaltered Muff-nonfits compared sidebyside with *DIO*'s 4 neat matches.

[3] Sociological background to such's inevitability (*DIO 4.3* ‡15 §G9): banishers are unwittingly *gambling* — risking their reputations irrevocably on the improvident demand that the pariah is *permanently valueless*. Since no blackballing archon can admit to jailing valid ideas, the exiled journal can't ever be credited for making a single discovery. So each time it does, its bet-redoubling shunners **must keep publicly faking its accumulating achievements' worthlessness** (welcome exception: ‡2's fn 10 on its eq.4). See, e.g., *DIO 4.2* ‡9 §T, *DIO 6* ‡3 §B2.

[4] Hipparchos' adopted solar orbit varied from time to time, as we saw at ‡2 §O.

[5] In temporal order, we call Trio A's eclipses: A1 (−382/12/22-23), A2 (−381/6/18-19), A3 (−381/12/12-13). Trio B analogously: B1 (−200/9/22-23), B2 (−199/3/19-20), B3 (−199/9/11-12).

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−146 gained or hired the scientific skills that ultimately made him justly famous. E.g., the *GD* latitudes of the sites near his Bithynian origins, Nicaea & Byzantion, latitudes which are too high by 1°1/2 & 2°, resp (Diller 1984 Table 15): astonishingly large errors, impossible for any transit instrument, esp. that which Hipparchos used from −146 on. (Rawlins *op cit* §K2 airs the possibility the errors arose not from vertical instruments but from use of horizon phenomena such as ortive amplitudes.) These latitudes almost certainly came into Ptolemy's *GD* from Hipparchos of Nicaea, who thus could not have been doing serious astronomy ere departing Bithynia (ultimately arriving at Rhodos by −146 at the latest). Again: if Hipparchos in −157 was fully conversant with instrumental astronomy, why did he need to indoor-obtain a −157 solst from Kallippos' calendar and use the resultant EH orbit (in whole or part) for years[13] (−157 to −145) to compute eclipses such as Trios A&B? The S.Solst used for final computation of the PH orbit was −145/6/26 3/4 (whether observed or extrapolated from observed −146/6/26 1/2), when the EH orbit gave way to PH — as we see from multiple coherent indicia: §L3, consecutive-triplet orbit-base (fn 13), −145 V.Eqx's capper PH-rôle (*idem*), and Physkon's −145 accession (§O3). Extra hint: eq.26's ultra(excessive!?)neatness. (Also: −145 minus −157 = 12^{y} = 0^{y} mod 4^{y}).

L5 From the −146/6/26 1/2 S.Solst, Hipparchos need only go back 11 Kallippic years, to create the "observed" −157/6/26 3/4 S.Solst of his *P.Fouad* 267A tables & could've even more easily extrapolated 1^{y} ahead to ensure a −145/6/26 3/4 S.Solst (if not confirmed by year-later outdoor re-observation) for establishing his ultimately canonical-regnal −145 PH orbit (for §L4). Pseudo-observed solstice-hours Kallippically extrapolated from his −146 solstice-hour for −157 & −145 would differ acceptably little from extrapolations based on Hipparchos' yearlength (eq.23 or eq.32): 53^{m} & 5^{m}, resp.

L6 As seen at Rawlins 1991W §§K4&8, Hipparchos was in −157 searching for a S.Solst not by outdoor observation but by indoor calculation. Which tells us that he at this time didn't know how to measure a solstice, nor even how to choose an expert who did. Perhaps it was just convenient (‡3 fn 6) to stick with the increasingly inaccurate Kallippic calendar, revered as that (too)long-standard among astrologers, most of whom ignore the outdoor sky, Hipparchos later becoming the 1st known major exception. This discussion occasions our tabulation of the indoor solstices we have from antiquity (Table 2, chronologically ordered according to date of creation), including Ptolemy's well-known 140 AD fraud at *Almajest* 3.1. The papyrus' Hipparchos solstice (2nd in Table 2) is only technically an indoor observation, as noted at fn 17: the accuracy of its outdoor procreator, the −146/6/26 1/2 S.Solstice (§M4), transferred faithfully (§L5) to the −157/6/26 3/4 extrapolation.

M Solving the −157 Double-Solstice Mystery

M1 The ultimate implication of the foregoing is weird but simultaneously satisfies the various[14] above-enumerated evidential features: following Hipparchos' outdoor capture of the −146/6/26 1/2 solstice, the papyrus' −157/6/26 3/4 solst was extrapolated from it

could allow an accurate solstice (§G1), but their sheer size (half a day!), and the proximity of their mean (12′) to the 16′ error characteristic of an asymmetric gnomon, suggest sufficient crudity as to cast doubt (independent of §L3) on whether he got an accurate outdoor S.Solstice ere Rhodos-arrival.

[13] No Hipparchos orbit until PH gibes with *Fouad*'s −157/6/26 3/4 S.Solst. ***But the PH orbit could not exist*** until the −145 V.Eqx. An orbit's 3 required empirical cardinal-pt bases were best arranged consecutively, and no Hipparchos Winter Solst was used for orbits. (Just for finding obliquity & latitude, as also 100^{y} earlier: Rawlins 1982G.) So the V.Eqx-S.Solst-A.Eqx triplet producing the final PH orbit used −145/6/26 3/4.

[14] Tihon 2010 p.7 proves col.3 adopted $-1^{d}/309$. Col.3's −657-epoch table was completed (−145) *before* computation of his then-still-incompletely-calculated eventual PH f-table, which (eq.32) rounded to $-1^{d}/300$ and used ϵ_{o} exactly fitting *Alm* 3.2 (§N1 item [5]) via the same PH yearlength. Did young Hipparchos use (§M2) epoch Phil 1 (−323) for astronomy while adopting epoch −657 for his astrological manual? — only later finally expanding back c.600^{E} from his time to Nab 1 for PH's f-table, which *effectively went back c.1200^{y} to c.−1350* for early eclipses: www.dioi.org/thr.htm#rbkv.

simply by subtracting 11^{y} of motion. (Thus replacing the awful EH solst, $-157/6/28\ 1/4$, in future editions of his horoscopic publications, such as the material used by the *P.Fouad* 267A astrologer.) Moreover, Tihon 2010 (p.2) found that (along with parallel columns for sidereal & "tropical" longitudes) *the papyrus' ephemeris retains a Kallippically-computed column of solar longitudes* (at quarter-century intervals) — startlingly consistent with Rawlins 1991W §K's proposals that [i] EH's mean solar motion was Kallippic and [ii] EH's foundation S.Solst was -157. Tihon discovered from the papyrus that its practical epoch[15] was $-657/2/4$ (Nab 90 Thoth 1), running 500^{E} (Egyptian years of 365^{d} each) and (like Ptolemy's *Handy Tables* & *Almajest* 6.3) at 25^{E} per line. Given the *Fouad*-astrologer's addition of mean solar motion for 21^{h} to an integral number of days from epoch, we know (since his horoscope is for 3 AM) his $-657/2/4$ epoch was 6^{h}.

M2 From these findings & his ultimate immortality (& *Fouad*'s citing "nativity" as its calculational purpose), we can guess Hipparchos had published an internationally popular, profitably-multiple-tradition astrological manual in -157, including a purely Kallippic table for mean solar longitude, eventually going 500^{E} into the past and perhaps 100^{E} more into the future: 600^{E} in all, possibly [vs §K1] the basis for Pliny 2.8.53's reference to Hipparchos' 600^{y} of calculations. The curious failure of the papyrus' (pre-*Almajest*) astrologer to cite any work later than -157 may indicate that Hipparchos' mature researches were more scientific than popular and were primarily intended for an astronomical not astrological audience. (Financed by selling horoscopes & manuals for? And-or gov't support?) When in -146 he realized how wrong the EH orbit's solstice was, he appended at least the column of mean solar longitudes based upon Metonic Y_{p} (eq.23). We may compute the Kallippic column's ϵ_{o} by working backwards from Kallippos' epoch (eq.1), when true solar longitude $\phi = 90°$ at $-329/6/28\ 1/4$, which (by PH's $e\&A$) is when mean solar longitude $f = 90°59'$. Result: the papyrus' middle (Kallippic) column's mean-longitude-at-epoch for $-657/2/4\ 6^{\text{h}}$ (§M1) was solar $\epsilon_{\text{o}} = 309°03'$ (vs actually $306°.7$).

M3 *Fouad* bears 3 columns of computed ϕ: [1] left,[16] [2] Kallippic or "mean" (middle), [3] Metonic "tropical" (right). The last is PH (but for eq.23's yearlength): we revolve back (again from $f = 90°59'$) for the $182767^{\text{d}}1/2$ from $-157/6/26\ 3/4$ to $-657/2/4\ 1/4$, finding $\epsilon_{\text{o}} = 308°56'$ ($49'$ for $14^{\text{h}}3/4$ klima). (This & §M2 rounded to $\epsilon_{\text{o}} = 309°$ for computing?)

M4 Finally, in answer to this section's semi-facetious titular question: the "$-157/6/26$" solstice was truly[17] outdoor-observed by Hipparchos at $-146/6/26\ 1/2$ & then — to replace his erroneous indoor epochal $-157/6/28\ 1/4$ solstice — he Kallippically-*reconstituted*[18] it back at $-157/6/26\ 3/4$, with but tiny concomitant error (§L5; Table 2) as he was fully aware. So, was Pliny 2.5.27 wrong in claiming that not even god can change the past?

N Statistical Impregnability of the −157/6/28 1/4 Solstice's Adoption

N1 To understand what *DIO* has accomplished here regarding Hipparchan solar theory, let us catalog the ***FIVE types of fits simultaneously achieved*** at Rawlins 1991W §§K&M:

[15] *Almajest* 3.1 shows that Hipparchos' solar observations were dated according to the number of years after "the death of Alexander" or equivalently epoch Phil 1, the ascension of Philip III: §O3.

[16] The $365^{\text{d}}1/4 + 1^{\text{d}}/(102\ 2/3)$ yearlength was far closer to the real anomalistic year (remainder: $+1^{\text{d}}/102$) than sidereal. Left-column yearlength is consistent with remainder $-7°1/2$ in ancients' key 345^{y} equation (Rawlins 1996C §C: implicit yearlength $\doteq 365^{\text{d}} + 1^{\text{d}}/100$), used in *Almajest* 4.2 to find ‡1 eq.4. *Fouad* not explicit whether left-column is for sidereal (Tihon 2010 p.6) or apsidal precession.

[17] The $-157/6/26\ 3/4$ solstice is not at all a fabrication. Hipparchos knew that extrapolating the $-146/6/26\ 1/2$ solstice to produce it would yield a datum differing but ordmag 1^{h} from the truth if his -146 observation was accurate. Hipparchos no more thought of extrapolation-reconstructing it as dishonest than he thought it a trick to find a solst by eq.5's interpolation. Neither resulting datum is a direct observation, but the procedure is scientifically proper and justifiable in both cases.

[18] One may hypothesize the reverse: indoor -146 solstice reconstituted from outdoor -157 solstice. But, aside from the question (§L4) of Hipparchos' crude instruments in -157: was getting-rich (fn 20) why he waited 12^{y} before adopting (fn 13) the $-145/6/26\ 18^{\text{h}}$ S.Solstice to found his PH tables?

P8 BM55555's yearlength $Y_{\text{U-M}}$ (eq.3), while not close to the mark, is the best of a poor lot; but no known ancient year-length was within 4^{m} of being correct. This, while Aristarchos' sidereal year-length was correct to ordmag 10 timesecs [Rawlins 2002A fn 15; Rawlins 1999 §§C8-C9] since the sidereal year is of no public interest [*ibid* §D3], so: no danger of an astronomical disaster like Meton's lunar&solar priesthood-peacepact (Rawlins 1991H fn 1), $-431/6/27\ 3/4$ kicking off the year *containing* the start of Greece's Great War $-430/4/4$-$403/4/25$. (Correlation un-noted in history-of-astronomy literature?) Meton's ploy launched a tradition: "innocently guilty" of preconception, observing scientists Aristarchos & mature Hipparchos were (Rawlins 1985S) cascadingly attracted to prior data that seemed to reconfirm eq.30's longago pax-Meton, which lay in wait for 600^{y} ere undoing non-observing non-scientist C.Ptolemy, who instead just computed his 140 S.Solstice from Hipparchos' Metonic calendar (as young astrologer Hipparchos had computed his original -157 SS from Kallippos').

Q Preconception's Wages: Hipparchos Neglects Kallippos' Solstice

Q1 The contention of Rawlins 1999 §D4 was that the tropical year-length estimates we have from antiquity (with the exception of eq.35) flock quite unrandomly around the artificial Metonic value of eq.31. These results vindicate Tobias Mayer's solution (modernly rediscovered by R.Mercier, K.Moesgaard, N.Swerdlow, & DR) of the source of the systematic error in the Hipparchos-Ptolemy solar tables, namely, the Hipparchos year mimicked the Metonic luni-solar yearlength: eq.31. So preconception from (evidently) near-universal belief in eq.31 caused Hipparchos to *miss the opportunity to acquire the 1st accurate tropical year-length*. Survey his career-long search for a trustworthy ancient-to-him solstitial anchor: [a] In -157, he uses Kallippos' -329 Summer Solst to anchor EH. [b] While 12^{y} later adopting Meton's eq.30, he observes the -145 S.Solst but finds it won't work Metonically (eq.31) with Meton's own -431 S.Solst unless (§C1) Meton's "start of day" is (falsely: eq.1) taken to mean dawn, thus his -12 -fudged -431 S.Solst anchors PH. [c] When, 11^{y} later, his new -135 S.Solst observation jars vis-à-vis the previous -145 one, he shifts anchor from Meton's -431 solst to Aristarchos' -279 solst for UH, in order to maintain (§Q1) his year-remainder at c.$-1^{\text{d}}/300$. [d] But for his ultimate anchor, Hipparchos never goes back to the only accurate solst of the now-known lot: Kallippos' *where he started* (item [a] above; or §K1). This takes us into the plainest proof of Metonic preconception's grip (§§P6&P8), & an obvious, previously-unasked question: ***why did Hipparchos never compare either of his outdoor solstices to Kallippos'***, whose S.Solst offered longer baselines than Aristarchos'. Had he done so for his 1st empirical solstice (-146), he'd have found (interval 183^{y}), treating seasonal hours naïvely:

$$Y_{\text{H1-K}} = \frac{66839^{\text{d}}1/4}{329-146} = Y_{\text{K}} - 1^{\text{d}}/122 \doteq 365^{\text{d}}.2418 \doteq 365^{\text{d}}1/4 - 1^{\text{d}}/122 \tag{36}$$

and, for his 2nd empirical solstice (-134), using an interval of 195^{y}:

$$Y_{\text{H2-K}} = \frac{71222^{\text{d}}}{329-134} = Y_{\text{K}} - 7^{\text{d}}/780 \doteq 365^{\text{d}}.2410 \doteq 365^{\text{d}}1/4 - 1^{\text{d}}/111 \tag{37}$$

These 2 potential (historically-unrealized) yearlengths' errors vs the real mean year (eq.28): $Y_{\text{H1-K}}$ $-1^{\text{m}}.0$, $Y_{\text{H2-K}}$ $-2^{\text{m}}.1$. And vs S.Solst yr (eq.29): $Y_{\text{H1-K}}$ $-0^{\text{m}}.1$, $Y_{\text{H2-K}}$ $-1^{\text{m}}.3$.

Q2 Despite solstices' failure to yield an accurate tropical year (due to truncations, prejudice for eq.31, & not choosing Kallippos' solstice as earlier anchor), solstices nonetheless contributed to gradual improvement of the solar orbit, being (§G1) the most reliable of the 4 cardinal points. Whatever the quality of the calendaric uses made of them, the 4 recoverable outdoor ancient solstices (Table 3) were so conscientiously accomplished by the methods we discussed at the outset (culminating in §J), that all four are accurate within their quarter-day rounding — rounding which (§O4) has made it impossible to tell whether the pre-rounded values were more than trivially in error. As noted at §O5, this is yet another vindication for the high level of ancient Greek science, and for those who've defended it.

Comparing his $-146/6/26\ 1/2$ solst with his hugely erroneous $-431/6/27\ 1/4$ dawn-version of Meton's solst ($-1^{\mathrm{d}}.2$ off: eq.4), $104095^{\mathrm{d}}1/4$ earlier, he found (for best *Almajest* 2.6 Athens-Rhodos klima $14^{\mathrm{h}}1/2$) PH-vs-Meton remainder $= -1^{\mathrm{d}}/300.66$; trivially rounding:

$$Y_{\text{P-M}} = \frac{104095^{\mathrm{d}} + [14^{\mathrm{h}}1/2]/2}{431 - 146} \doteq 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/300 \doteq 365^{\mathrm{d}}.24667 = Y_{\mathrm{H}} \qquad (32)$$

This is the 1st time a modern has empirically justified by calculation astronomer Hipparchos' famous yearlength Y_{H}, adopted by Ptolemy and used for centuries thereafter. From here, Hipparchos devised his ***astonishing Great Year vision with its 5-stage geometrically embedded integral-return cycles*** ($304^{\mathrm{y}}1/4$, $608^{\mathrm{y}}1/2$, 1217^{y}, 2434^{y}, 4868^{y}), fully unfurled at Rawlins 2002A fnn 14, 16, 17. This Great Year fixed his long-view yearlength Y_{G}:

$$Y_{\mathrm{G}} = 365^{\mathrm{d}}1/4 - \frac{1^{\mathrm{d}}}{304\ 1/4} = Y_{\mathrm{K}} - \frac{16^{\mathrm{d}}}{4868} \doteq 365^{\mathrm{d}}.24671 \qquad (33)$$

We note that period 304^{y} (which is exactly 4 Kallippic cycles and 16 Metonic cycles) is clearly attested for Hipparchos by Censorinus (Heath 1913 p.297); for 4868^{y}, see fn 10 [4].

P5 But then, 12^{y} later, along came Hipparchos' $-134/6/26\ 1/4$ S.Solst, 5^{h} earlier than predicted by the PH orbit. (For potential effect, compare eq.35 to eq.32!) So did Hipparchos switch to a new year-length value? No — he instead (like the conservatism of §M1) also chose (*Alm* 3.1) a *near-equally* PH-discordant (Rawlins 1991H §B5) prior S.Solst, that of Aristarchos of Samos (eq.2; Table 1; Rawlins 1991H eq.8), $-279/6/26\ 1/2$, such that the new equation paralleling eq.32 gives near-enough the same PH mean motion F, but now from finding that seasonal $52960^{\mathrm{d}}3/4$ in the 145^{y} Aristarchos-Hipparchos gap yields:

$$Y_{\text{H-A}} = \frac{52961^{\mathrm{d}} - [14^{\mathrm{h}}1/2]/2}{279 - 134} \doteq Y_{\mathrm{K}} - 1^{\mathrm{d}}/263 \doteq 365^{\mathrm{d}}.24619 \qquad (34)$$

(Remainder was $-1^{\mathrm{d}}/290$, if Hipparchos computed $Y_{\text{H-A}}$ without accounting for seasonal hours.) The intent to somehow roughly justify preserving his original PH orbit's year-length (eq.32) is obvious. This evaded (see similarly at ‡3 fn 6) recalculating-replacing his PH solar mean motion tables, based on eq.32. That he adopted $Y_{\mathrm{H}} = 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/300$ is clear from his own words (quoted at *Almajest* 3.1). Y_{H} exactly underlies PH' ϵ_{o} & table of mean solar motion F (§O3; *Almajest* 3.2).

P6 But an admirably independent party dissented from locked-in Metonic (eq.30) rigidity, as we know from Babylonian cuneiform text BM55555 (ACT 210, c.100 BC), the 1st Babylonian record provably based (Rawlins 1991H) upon Greek astronomers' work. (A text also containing the "Babylonian" month, likewise based on Greek research: Rawlins 2002A & Rawlins 2002U.) BM55555 bears a schismatic year-length (eq.3 & Rawlins 1991H §§A1-A2), which was anciently found (eq.4) by comparing Hipparchos' -134 solstice to Meton's solstice (instead of Aristarchos'), a 108478^{d} interval over 297^{y} (eq.3):

$$Y_{\text{U-M}} = \frac{108478^{\mathrm{d}}}{431 - 134} = Y_{\mathrm{K}} - \frac{1^{\mathrm{d}}1/4}{297} = Y_{\mathrm{K}} - \frac{1^{\mathrm{d}}}{237\ 3/5} \doteq 365^{\mathrm{d}}.2458 \qquad (35)$$

— the most accurate (see §P7) of the poor (truncation-corrupted) anciently-adopted tropical year values that have come down to us. (Actual mean year-length then was $365^{\mathrm{d}}.2425$: eq.28.) It is possible that Hipparchos flirted with using $Y_{\text{U-M}}$ (or published it in one of his many lost works without using it any orbit that we have), but all extant records indicate that Hipparchos stuck with eq.32's remainder (or a close approximation thereto).

P7 After our extensive discussion (§§F-J) of how ancients found solstices, it is disappointing that [a] so few accurate ones survived for us, and [b] the calendaric intent was so consistently vitiated by truncation, which neatly (§E3) led to apparent repeated-confirmation of the delusion that Meton's seriously-inaccurate effective-equating of the tropical year with 235/19 months (eq.30), was valid. We now list the above three ancient year-length-estimates' errors vs the actual mean year (eq.28): Y_{HM} $+6^{\mathrm{m}}.2$, Y_{H} $+6^{\mathrm{m}}.0$,[24] $Y_{\text{U-M}}$ $+4^{\mathrm{m}}.7$. And vs S.Solst yr (eq.29): Y_{HM} $+7^{\mathrm{m}}.1$, Y_{H} $+6^{\mathrm{m}}.9$, $Y_{\text{U-M}}$ $+5^{\mathrm{m}}.6$.

[24] The $+6^{\mathrm{m}}/1^{\mathrm{y}}$ excess of Hipparchos' Metonic year over reality (eq.28) produced solar mean longitude error $-1^{\circ}.1$ by Ptolemy's time, thus (Thurston 1998A §S2) revealing Ptolemy's faked "observations".

[1] The -157 EH orbit fits the usual 3 cardinal pts, which are either attested (-157 A.Eqx) as Hipparchos' or extrapolatable (krikos -157 V.Eqx) from data cited by him, or taken exactly (-157 S.Solst) from a famous calendar repeatedly used by him (*Almajest* 3&5-7).
[2] A valid EH orbit must (within c.$1'$: *ibid* §K11) place the solar true longitude ϕ at 00° for [1]'s V.Eqx, at 90° for [1]'s S.Solst, at 180° for [1]'s A.Eqx *all at the $1^{\mathrm{d}}/4$ precision Hipparchos always uses* (*Almajest* 3.1). The PH orbit is already known (Neugebauer 1975 p.58) to satisfy such a like condition. See statistical exploitation of this point at §N3.
[3] For each trio, the proposed orbit must fit *Almajest* 4.11's true longitude ϕ-intervals:
The EH orbit (Rawlins 1991W §K) fits both Trio B intervals within ordmag $1'$.
The EH-PH-meld Frankenstein orbit (*ibid* §M) fits both Trio A intervals within ordmag $1'$.
[4] The solar mean motion must be reasonable, not conjured-up at convenience. Trios A&B both fit the already famous Kallippic solar motion. Additionally, thanks to Anne Tihon, we now have (for the 1st time) *direct* evidence that Hipparchos used Kallippic motion (it's right at *P.Fouad* 267A's middle column), as earlier 1st hypothesized at Rawlins 1991W §§K&M.
[5] Each ϵ_{o} must be convincing. For Kallippic solar speed, Trio B's strikingly integral Phil 1 ϵ_{o} ($228^{\circ}00'$) exactly fits the Kallippic calendar's founding S.Solst, $-329/6/28\ 1/4$. The Frankenstein orbit's $\epsilon_{\mathrm{o}} = 227^{\circ}2/3$ exactly fits (via PH's $365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/300$: eq.32) the Nab 1 Thoth 1 $\epsilon_{\mathrm{o}} = 330^{\circ}3/4$ of the standard *Almajest* 3.2 solar mean motion table built upon the PH orbit which supplies the *constants*-half of the hybrid (§L3) Frankenstein orbit's elements ($A = 65^{\circ}$ & $\epsilon_{\mathrm{o}} = 227^{\circ}2/3$), the other half being the *tabular* elements of the EH orbit ($e = 3^{\mathrm{p}}1/4$ & $Y_{\mathrm{K}} = 365^{\mathrm{d}}1/4$), a dichotomy explained here at §L3 & ‡3 §E3.

N2 In the effectively unrefereed[19] *JHA*, one of its board members volunteers to be the sole critic publicly rejecting the EH orbit, contending (fn 10) that *DIO*'s analysis has not established unique multi-fits. *He can prove his contention anytime by coming up with alternate orbits (distinct from ours) which also neatly satisfy §N1's five conditions*.
[As of 2018, our challenge has not been met. And never will be.]

N3 A Funny Thing Happened on the Way to the Dumpster. Our cynosure Muffia's wisdom has decreed (‡3 fn 8) that anything gotten from the allegedly loose *Almajest* 4.11 eclipse trio-intervals is worthless. When crazy Rawlins 1991W claimed *previously unheard-of* Hipparchan use of [1] a -157 S.Solst & [2] Kallippic solar motion, could be extracted from the eclipse intervals, all right-thinking JHADists knew better: just throw out all that *DIO* junk. But then, over 10^{y} later, a funny thing happened: the miraculous, 1-in-a-million finding of an ancient papyrus on the subject. And — (*don't tell anybody*) you know what it said? *It testified that Hipparchos had used [1] a -157 Summer Solst & [2] Kallippic solar motion*.

N4 Even funnier: an unbigoted community wouldn't need papyrus-confirmation. Check the several speed-bumps that should've slowed *JHA* ere reaching the Orwellian dumpster it would burn vital research in: [a] The coincidence that *both* Worthless trios happened to mutually confirm Hipparchos' use of Kallippic motion. [b] The *a priori* improbability of so many fits *accidentally* flowing from Rawlins 1991W §K's spare premises is obvious. [c] Contra *JHA*'s careless contempt (§N2) for scrupulous and multi-expert-checked (fn 9) research, *ibid* fn 205 showed in *JHA*-uncited detail that *Almajest* 4.11's intervals set narrow limits on elements. [d] If *JHA* is right, then the fact, that our EH orbit (mathematically-consistent with *Almajest* 4.11's data) fits on-the-nose all 3 quarter-day cardinal-points (§N2 condition [2]), ***is just pure-coincidental luck***. How lucky? Of Rawlins 1991W's three EH cardinal points (Hipparchan orbit-math avoided W.Solstices: fn 13), all occur within $\pm 20^{\mathrm{m}}$ (solar motion under $1'$) of a quarter-day marker (00^{h}, 06^{h}, 12^{h}, or 18^{h}): $\phi = 00^{\circ}$ at $-157/3/24$ 11:41, 90° at $-157/6/28$ 6:06, 180° at $-157/9/27$ 11:42. The chance probability of 3 hits within 20^{m} is 1 in nearly 8000. Note: the universally-accepted *Almajest* PH orbit fits much less well (than EH): V.Eqx at $-145/3/24$ 5:13; or 5:26, for $A = 65^{\circ}$.

[19] See www.dioi.org/jha.htm#pdfr. *JHA* Pb (!) paper Duke 2008W displays not only the mistake just analysed but 3 others equally obvious to nonzombie refereeing: 2 at §B4 & 1 more at fn 10's final line.

N5 Until −145 Hipparchos was demonstrably (§L3) using the EH orbit and thus the co-foundational Kallippic 6/28 1/4 S.Solstice consistent with it — ***but not at all with 6/26 3/4***. The historic 1st PH calculation, narrowly datable (§O3) to the 4 weeks between −145/3/24 V.Eqx & −145/4/21 eclipse, presaged the centuries-durable PH orbit through altering (vs the EH orbit) V.Eqx by $-1^{\rm d}/4$, S.Solst by $-1^{\rm d}1/2$, A.Eqx by $-1^{\rm d}/4$.

O Recovering Hipparchos' Lost −146 Solstice

O1 From −145 to −134, Hipparchos' mainstay solar orbit was PH, later appropriated (essentially unaltered) by Ptolemy (*Almajest* 3.2&6), standard among astrologers for centuries, cited as "perfect" by Julian the Apostate (1:429), $500^{\rm y}$ later, though by then differing from reality by $2^{\rm d}$ or 2°! Rawlins 1991H §§C-D showed that in −134 Hipparchos abandoned PH and adopted the superior UH orbit. But the question that has never previously been answered (or even asked) is: whence came the S.Solst needed for the PH orbit?

O2 The only 2 years *Almajest* 3.1's Hipparchos cardinal-point data lists both equinoxes: −146 AE to −145 AE & −142 VE to −141 VE, the latter barred by its A.Eqx's discord with the PH orbit. His 1st outdoor-observed S.Solst cannot be −145/6/26 3/4 since Hipparchos used (Rawlins 2009E §B5) the PH orbit months earlier to place the mid-eclipsed Moon (−145/4/21), so the −146/6/26 1/2 S.Solst was his 1st Rhodos sky-record.

O3 *Almajest* 3.1's collection of Hipparchan cardinal-point observations cites only Autumn[20] Equinoxes before his −145/3/24 V.Eqx capped capture (§O2) of his 3rd Rhodos solar cardinal-pt data, of the three needed to compute (like Neugebauer 1975 pp.58-60) his PH orbit, just in time to figure mid-eclipse for his −145/4/21 measure of Spica's place (*Almajest* 3.1). [Added 2018/2/10. The timing suggests: did he move to Rhodos for its good weather just before the −146 S.Solst, partly to ensure that he wouldn't miss measuring the −145 eclipse?] *Almajest*'s PH orbit (epoch Phil 1 Thoth 1 = −323/11/12 Alex App Noon; elements at Rawlins 1991W §K10) gives solar true longitude $\phi_{\rm AE}$ for his −145/9/27 1/4 A.Eqx, only $2^{\rm d}1/4$ before the regnal epoch Ptolemy Physkon 1 Thoth 1 (Toomer 1984 pp.11&133), with PH mean anomaly $g = 116°2/3$ (*Almajest* 3.7; Neugebauer 1975 p.59):

$$\phi_{\rm AE} = 227°2/3 + \frac{360° \cdot 64967^{\rm d}3/4}{Y_{\rm H}} - \arctan\frac{\sin 116°2/3}{24+\cos 116°2/3} = 180°00'00'' \quad (26)$$

It appears[21] that the PH solar mean-longitude-at-epoch (same as Ptolemy's at *Almajest* 3.2) $\epsilon_{\rm o} = 227°2/3$ was set by Hipparchos to ensure the exactitude of eq.26, consistent with the PH orbit's launch upon −145 V.Eqx's capture. So we have traced the A.Eqx-origin of Ptolemy's hitherto-unexplained Nabonassar 1 Thoth 1 $\epsilon_{\rm o} = 330°45'$ (*Almajest* 3.2&7), $424^{\rm E}$ prior to Hipparchos' Phil 1 epoch. Hipparchos thus gave calendaric priority to the A.Eqx (fn 20). Anyway, it's obvious that −146 SS to −145 AE (§O2) was the period of the 3 observations used to found the PH orbit (Rawlins 1991H §C8 or Rawlins 1991W §K10); which allows us to reconstruct the previously unknown real outdoor Hipparchos S.Solst that co-launched PH (and, as already seen at §M1, retro-created the papyrus' −157 solst):

$$-157/6/26\ 3/4 + 11 \cdot (365^{\rm d}1/4) = -146/6/26\ 1/2 \quad (27)$$

Adding this new find to those cited at §C2, we have four genuine outdoor ancient solstices. Again: none's hour is unambiguously cited in extant material. All are *DIO* reconstructions.

O4 There is no proof in Table 3 that observational error exceeded even a fraction of an hour, since rounding to $1^{\rm d}/4$ precision could account for most of the O−C error.[22]

O5 We have already (§D) considered how well previously-known real Greek solstices support *DIO*'s steady (from *DIO 1.1* ‡1 fn 24, 1991) contention that ancient science was far more empirical and competent than longtime orthodoxy has recognized. The −146 S.Solst just adds further confirmation to what in a sane scholarly community would have long since been incorporated — to its intellectual (and reputational) profit.

P PH Yearlength's Origin? Hipparchos' Ingenious Great-Year Cycle

P1 From his −145 V.Eqx, S.Solst, & A.Eqx, Hipparchos computed (method: *Almajest* 3.4; Rawlins 1991H §C) three of his final PH orbit's elements: $\epsilon_{\rm o}$, e, & A. But the 4th of the required 4 elements, the mean motion F, must depend in part upon earlier astronomers' observations. Except for hist.astron's dearest archons, scientific historians know how ancients estimated year-lengths: by comparing solstices centuries apart.

P2 In order to gauge ancient solstices' and year-lengths' accuracies, we need to know the actual values at that time. For Hipparchos' era, the true mean tropical[23] year was (Rawlins 1999 §C10) about $365^{\rm d}.2425$:

$$\text{Actual Hipparchos-Era Tropical Year-Length} \doteq 365^{\rm d}.2425 = 365^{\rm d}1/4 - 3^{\rm d}/400 \quad (28)$$

The foregoing rounding happens to be equal to Jesuit Christopher Clavius' Gregorian-rule year-length, established 17 centuries later (when the year was nearer $365^{\rm d}.2423$), and which we live-by today. (See puzzle at *DIO 4.2* p.2, instantly solved by K.Pickering & R.Freitag.)

P3 And there is an extra factor which is oft-forgot, namely (fn 23): each of the four cardinal-points has its own year-length — generally differing from the others by a few ten-thousandths of a day. Both their relative proportions and their absolute lengths vary secularly. For the present discussion, we should know the Hipparchos-era S.Solst value:

$$\text{Actual Hipparchos-Era S.Solst Year-Length} = 365^{\rm d}.2419 \quad (29)$$

To measure empirical error, compare ancient figures to eq.29; to measure vs mean year-length (which ancients thought they were determining), compare to eq.28.

P4 As soon as his −146 S.Solst measurement was in hand, Hipparchos returned to his earlier dabbling with Meton, which had led (§K5) to a yearlength tantalizingly close to compatibility with Meton's definition from his ratio (still used for modern Easter):

$$\text{Metonic Year } Y_{\rm M} = 235 \text{ months}/19 \quad (30)$$

which (via ‡1 eq.4) requires

$$Y_{\rm HM} = (235/19) \cdot 29^{\rm d}31'50''08'''20'''' \doteq 365^{\rm d}1/4 - 1^{\rm d}/314.7 \doteq 365^{\rm d}.24682 \quad (31)$$

[20] Or did Hipparchos have an unusual calendaric interest in the Autumn Equinox, since it was near the Egyptian calendar's start (Thoth 1) in his era? During the year of his UH-founding −134 S.Solst (eq.4), his −134/9/24 A.Eqx occurred smack-on Thoth 1 (Rawlins 1991H fn 14); and his UH solar mean longitude 180° occurred at $10^{\rm h}$ on Thoth 1 during the UH orbit's −127/9/24 epoch-day (*ibid* eq.28). During the $11^{\rm y}$gap 'twixt Hipparchos' −157 & −146 observations, did astrological tables' (Tihon 2010) sales make him rich enough to return to creativity (Rachmaninov [www.dioi.org/rar.htm] parallel 1917-1926)? — moving to clear-skied Rhodos, to facilitate fulfilling a dream of founding astronomy empirically.

[21] Note: −145 S.Solst proposed for ultimate PH orbit on thin evidence as early as Rawlins 1985H. And see Rawlins 1991W §M6, where it is also noted that −145 was ***a regnal year***, Ptolemy VII Physkon's. See Rawlins 1991H fn 7 for Physkon 1 Thoth 1, which usefully clinches −145 as PH's epoch, crucially since eq.26 adjusted for other nearby years would be nearly as well-fitting for A.Eqx; −145's V.Eqx is $+1'.9$ off & S.Solst off $+0'.7$. (Rawlins 1991W §M4's best Frankenstein-orbit fit was for $A = 65°$, but $A = 65°1/2$ fits Trio A's data nearly as well & it's the Hipparchan apogee preserved even centuries later at *Almajest* 3.7: Neugebauer 1975 pp.58f. The superb analysis of van Dalen 1994 showed that the *Almajest* 3.6 anomaly table's numbers were actually generated from $A = 66°$.)

[22] To consider an extreme case: if a S.Solst that occurred at 14:00 were measured by the observer as having occurred at 15:01, which he accurately rounded to traditional $1^{\rm d}/4$ precision (i.e., to $18^{\rm h}$), an O−C error of merely $1^{\rm h}$ would effectively quadruple, appearing to us to be a $4^{\rm h}$ O−C error. See fn 8.

[23] Technically, what has long been called a "tropical year" is a misnomer, since it refers to the sidereal year minus the effect of precession. But that standard figure — eq.28 — was not (§P3) the same as either of the two solstitial years: i.e., the mean Sun's returns to the Summer Tropic & Winter Tropic. Nor the same as the years measuring the mean Sun's returns to the Vernal & Autumnal Equinoxes. (You'll have to ask the esteamed *Journal for the History of Astronomy* about the Winter Equinoctial Year: §B1.) Note that in antiquity the average of the years of the S.Solst&W.Solst virtually equalled eq.28, as did the average of the V.Eqx&A.Eqx years.