# ‡2 Hipparchus and Spherical Trigonometry

**by Curtis Wilson**[1]

## A Hipparchus' Trigonometric Equation on the Sphere

That spherical trigonometry was developed or used by Hipparchus has occasionally been claimed.[2] According to Neugebauer, however, the solution of spherical triangles became possible only with the discovery of two theorems by Menelaus (first century A.D.).[3] The sole evidence adduced to the contrary that I am aware of is Hipparchus' alleged use of a formula to determine latitude from length of longest day:

$$\tan\phi = \frac{-\cos\frac{M}{2}}{\tan\epsilon} \tag{1}$$

where $\phi$ is latitude, $M$ is the length of the longest day converted to angle at 15° to the hour, and $\epsilon$ is the obliquity of the ecliptic.

## B The Analemma Alternative

**B1** However, the Greek equivalent to formula 1 is derivable, without use of spherical trigonometry, by analemma methods. That Hipparchus used such methods seems very likely indeed. In his *Aratus Commentary*, he claims to have derived "by rigorous methods" (δια των γραμμων) the arc above the horizon of a star of declination 27;20°, for latitude $\phi$ = 36°.[4] The problem in effect uses formula 1 backwards, with declinaton 27;20° replacing the obliquity $\epsilon$, and $M$ as the unknown. Hipparchus' result was 224;15°; a present-day hand calculator gives 224;07°.[5]

**B2** To derive the Greek equivalent of (1), we use the analemma construction shown in Figure 1, which is adapted from Neugebauer's Part I Fig.284.[6] $OU$ is the trace of the horizon plane, $OM$ the trace of the equator, $\epsilon$ the obliquity of the ecliptic, $\phi$ the latitude. The half circle $VST$ represents half the Sun's path on the day of the summer solstice, with

[1] [Note by DR.] Curtis Wilson (St. John's College, P.O.Box 2800, Annapolis, MD 21404) is rightly respected as one of the world's leading experts on Enlightenment-period mathematical astronomy. He is co-Editor of the *General History of Astronomy* (the majority of whose 4 Editors are not admirers of DR). The present contribution marks the 1st submission to (and, of course, publication by) *DIO* of a pure-Neugebauer-Muffia-viewpoint article. As ever (*DIO 4.2* ‡7 §B43), we encourage the submission of others. [Paper printed essentially as received. Headers & bio supplied by DR.]

[2] I. Thomas, *Dictionary of Scientific Biography 13*, 319; D. Rawlins, "An Investigation of the Ancient Star Catalogue," *Publications of the Astronomical Society of the Pacific 94* (1982), 368; D. Rawlins *DIO 4.2* (1994), "Competence Held Hostage #2". [Note by DR. For dissent on the contended question, see *Vistas in Astronomy 28*:255 (1985) n.9 (van der Waerden), *DIO 1.2-3* fn 38 & §§G2, P1, & P2, *DIO 2.1* ‡3 §A2, *DIO 4.1* ‡3 §§D & E5-E7 and fn 17, *DIO 4.3* ‡14, *DIO 6* ‡1 §G5.]

[3] O. Neugebauer *A History of Ancient Mathematical Astronomy* (hereinafter *HAMA*; Springer-Verlag, 1975), pp.26-29, 301.

[4] Neugebauer, *HAMA*, 301-302.

[5] Carrying out the calculation by the Greek formula (formula 5 below), and using linear interpolation in G.J.Toomer's reconstruction of Hipparchus' table of chords (*Centaurus 18* (1974), 8), I obtained the result 224;08°.

[6] *HAMA*, 1310.



$S$ the point where the Sun sets; this half circle, originally at right angles to the plane of the diagram, has been swung about $TV$ through 90° so as to lie in the plane of the figure. The arc $ST$ therefore represents half the longest day, or $M/2$, and the angle $n$ shown in the diagram is half the excess of $M/2$ over 90°. Then

$$VT = 2r_d = \text{crd}(180^\circ - 2\epsilon) \tag{2}$$

$$UR = \frac{r_d}{2}\text{crd}2n \tag{3}$$

and

$$RO = \frac{\text{crd}2\epsilon}{2} \tag{4}$$

so that

$$\frac{UR}{RO} = \frac{\text{crd}2\phi}{\text{crd}(180^\circ - 2\phi)} = \frac{r_d\text{crd}2n}{\text{crd}2\epsilon} \tag{5}$$

Given that half the chord of the double angle is equal to the sine, and $\sin n = -\cos(90^\circ + n)$, then (5) transforms into (1).

## C DIO Position Not Justified

If, then, Hipparchus' use of (5) is the basis for the claim that he had spherical trigonometry at his command, the claim is unwarranted.

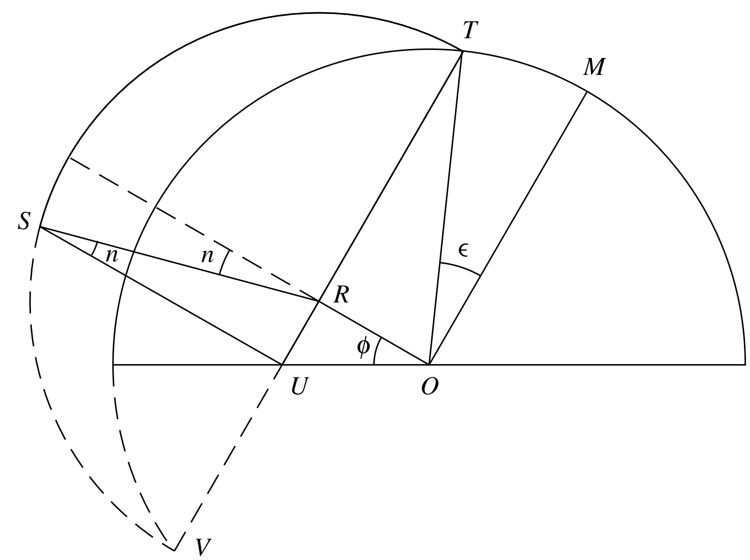


Figure 1. Rendition by KP & DR.