‡1 Aristarchos & the “Babylonian” System B Month

The Empirical and Calendaric Bases of Ancient Astronomy’s Prime Parameter

by Dennis Rawlins

A Derivation from the 345 Year Cycle & Aristarchos’ Great Year

A1 Greek civilization achieved technological superiority over Babylon — even conquering & permanently occupying the city (fn 2 [f]) — ordmag a century before our earliest records of the famous and highly accurate (§A3 [b]) System B “Babylonian” month:

$$M_{\mathrm{A}} = 29^{\mathrm{d}}31'50''08'''20'''' = 765433^{\mathrm{d}}/25920 = 765433^{\mathrm{h}}/1080 \qquad (1)$$

Current orthodoxy has been assuming that eq.1 is due to Babylon. But, using Greek relations (empirical & conventional), the following[1] will trace a very few steps of plain arithmetic leading from Greek relations (eqs.6&7) precisely to eq.1. This will be accomplished on the theory that eq.1 was due to the daring Greek astronomer and innovator, Aristarchos of Samos, the earliest scientist to teach heliocentrism widely (long-persisting ancient influences of which are noted at Rawlins 1987 p.238 & nn.34-38, Rawlins 1991W eqs.23&24, & Rawlins 1991P). The Aristarchan connexion can be made more swiftly, fully, & exactly than is possible for any Babylonian-data-based explanation (even though extant Babylonian data outnumber Aristarchan data by a factor of thousands). And we’ll enjoy a series of reality-checks along the way to discovering and independently confirming the origin of this, the most central of all ancient astronomical parameters.

A2 The sole empirical foundation upon which one could firmly base a monthlength as remarkably accurate as eq.1, is the very same one which Ptolemy (from Hipparchos) cites

[1] [The present paper has evolved for over two decades. (The derivation of M_{A} from a saros-Kallippic calendar [without realization yet of Aristarchos’ involvement] initially appeared in eqs.69-70 of a 1983 DR ms, which Gingerich suppressed at that time and van der Waerden was 1st to appreciate: *DIO 7.1* ‡6 fn 3. (Rest of paper now published: Rawlins 2003J.) Some of the 2nd half of the paper (culminating in eq.12’s match to eq.13) was likewise suppressed (also 1983, for an “indefinite” period, by the *JHA*’s M.Hoskin [MH to DR 1983/3/21] — details at Rawlins 1999 §F [and Rawlins 1991W §B3]). Its key findings eventually appeared in this journal: e.g, *DIO 1.1* ‡6 fn 1 & *DIO 9.1* ‡3.) Thanks to the encouragement of Christopher Walker and John Steele, the reconstruction was first delivered in more mature form on 2001/6/27 at the British Museum, as part of the conference *Under One Sky*, and was then published (condensed & simultaneously with the present edition) at *Alter Orient und Altes Testament 297*:295. During the writing of this paper, the author has repeatedly benefited from the learned input of Christopher Walker, Alexander Jones, John Britton, John Steele, and Hugh Thurston.]

as eq.1’s source, the precious Greek[2] 345^{y} eclipse period-relation (*Almajest* 4.2),[3] which was (and is) of striking constancy in duration (to appreciate this point fully, compare to ‡2 §§B7&B8), due to virtually integral returns in synodic and anomalistic (lunar & nearly solar) revolutions.

$$4267^{\mathrm{u}} = 4573^{\mathrm{v}} = 345^{\mathrm{g}} - 7°1/2 = 4630^{\mathrm{w}}1/2 + 11° = 126007^{\mathrm{d}}01^{\mathrm{h}} = 3024169^{\mathrm{h}} \qquad (2)$$

(Abbreviations adopted throughout: d = days, h = hours, u = synodic months, v = anomalistic months, w = draconitic months, g = anomalistic years.)

A3 Cutting past myriad confusing perturbations, this neat relation handed its discoverer (who could have found it by simple arithmetic centuries before Aristarchos, especially [see fn 7] after c.400 BC): [a] Realization of the stability possible in celestial mean motion (historically of high import: Rawlins 1996C fn 18) — this cycle may even have helped initiate the entire concept of mean motion. [b] A highly accurate value for the synodic monthlength, correct then and now to a fraction of a timesec (*ibid* fn 12). [Previous attempts to explain this accuracy have openly admitted failure. See, e.g., n.21 of Britton

[2] Several contextual points (additional to those cited elsewhere here, e.g., chronology: §A1 & fn 14) add to this paper’s mathematical reconstruction, in further suggesting a Greek origin for eq.1: [a] The attestation (§A2) of its empirical basis (eq.2) is purely Greek. (Or, to mirror certain Babylonianists’ obsession with text-paucity: there’s not-a-jot of evidence that Babylonians knew anything about the 345^{y} cycle that underlay “their” monthlength. Had Babylonian wisdom been at work, a highly accurate M would have existed in Babylon long before Greece conquered it. [See §A6, Rawlins 1991W fn 81 & Rawlins 2003P §H.]) [b] Other than eq.4, every equation in §A is virtually equal to the others; but the oldest attested one is eq.6 — *which unquestionably was used by Aristarchos* (§A7). [c] There are several unsubtle indications of early Greek mathematical use of eclipses. See Rawlins 1985G pp.264-265, Rawlins 1991H fn 1, and Rawlins 1996C fn 34. Scientific Greek lunar observations — more sophisticated than mere eclipse-recording — certainly go back at least to c.300 BC (Timocharis): *Almajest* 7.3. The fact that the famous −330/9/20 Arbela eclipse was the last visible before the epoch of Kallippos’ lunisolar calendar suggests that eclipses figured in Greek astronomical math well before Aristarchos. Our main Greek mathematical-astronomy source, Ptolemy, mentions no other Greek eclipses in this period — but, then, the *Almajest* cites no eclipses at all (Greek or Babylonian) between −381 and −200. (The not-a-jot contingent would leap all over this — except that a different Greek work cites [*GD* 1.4.2] the −330 Arbela eclipse for mathematical purposes. See ‡2 fn 7.) [d] The 1 hour remainder in eq.2 smells more of Greeks than Babylonians, since the latter’s day-division was via degrees ($1^{\mathrm{d}}/360$), arcmin, arcsec, etc — while Greeks used our modern conventional hour (though Aristarchos’ time-units aren’t known): $1^{\mathrm{d}}/24$, the very amount of Greek-attested eq.2’s revealing remainder. (Greek-hour rounding eases our path to M_{A}. Since the actual average ancient 4267 month span exceeded $126007^{\mathrm{d}}01^{\mathrm{h}}$ by a few tenths of an hour, a Babylonian expression for this time-span would have likely been $126007^{\mathrm{d}} + \mathrm{c.}20°$, which by eq.3 would’ve produced a value for M discrepant with M_{A} already by the 3rd sexagesimal place: 9 instead of 8. [But *note well*: Britton’s §A8 theory eliminates the problem.] [e] The most famous among the extremely small number of Babylonian tablets explicitly providing the “Babylonian” monthlength M_{A} is ACT 210 (BM 55555); but it is now generally acknowledged that the adjacent yearlength (on that very same 100 BC tablet: Rawlins 1996C fn 16) is of Greek origin, as discovered at Rawlins 1991H §A6. [f] By the time M_{A} is known to have been used, Babylon had been ruled by Greece for many decades. (See Rawlins 1991W §E3 & fn 73, *DIO 4.2* ‡9 §K9, Rawlins 1996C fn 128.) Are Babylonianists not implicitly contending that subjugation somehow made Babylonian astronomers more original & accurate than ever?

[3] A few of Ptolemy’s most invincibly innocent and humorless admirers have repeatedly objected to my educational rendering of “Almagest” as “Almajest”. Comments: [a] Toomer 1975 p.187 reasonably suggests that “Almagest” means not The-Greatest (as is commonly believed) but just the “big” astronomical collection (in contrast to Pappos’ well-known little astronomical collection). Thus, the term may be no more a commendation than is the title of Schubert’s “Great” Symphony-in-C, where the “Great” is supplied merely to distinguish this massive work from his earlier little Symphony-in-C. [b] It was the Arabs who contracted “the great compilation” by Ptolemy into a single word, but their rendition was “al-majistī ” — later corrupted via Latin into the present spelling with a “g”. (All explained at Toomer 1975 *loc cit*.) So, I trust that there will be no further complaint at my adoption of the “j” from the original Arabic contraction. [See *DIO 1.1* ‡1 fn 6.]

1999 (‡2 Refs).] (The anomalistic monthlength V determined by eq.2 is also quite accurate: good to about 1 timesec.)[4] The monthlength eq.2 provides is (see Copernicus 1543 [4.4]):

$$M_e = 126007^d01^h/4267^u = 3024169^d/102408 = 29^d31'50''08'''09'''' \ldots \quad (3)$$

In Aristarchos' era, the interval between an eclipse and its 345^y-ago partner was always within about an hour of 126007^d01^h with an rms scatter (around a value a few tenths of an hour higher) of less than a half hour ($0^h.4$). So the error in eqs.2&3 was merely 1 part in ordmag 10 million.

A4 The Kallippic yearlength (see fn 5), basis of the Aristarchos circle's Dionysios calendar (van der Waerden 1984-5; *DIO 1.1* ‡1 fn 23), was Julian before Caesar:

$$Y_K \equiv 365^d1/4 \quad (4)$$

(Question-in-passing: why launch the Dionysios calendar at all? — since Kallippos' calendar had the same yearlength. Speculative answer: the Dionysios calendar incorporated *additionally* Aristarchos' typically large-scale-visionary Great-Year [§A7, fn 14]. He was famous in antiquity for proposing the largest universe, too: see Archimedes' *Sandreckoner*.)

A5 Eqs.3&4 determine the number of Kallippic years in each M_e-based 223-month saros (where we will henceforth use superscript K for Kallippic years):

$$223M_e \approx 18^K + 10°40'00''.3 \quad (5)$$

A6 Reality-check: eq.5's remainder is astonishingly close to that in Aristarchos' (§A7) well-known saros equation (*Almajest* 4.2) — a relation which constitutes a prime instance of Greek scientists' clever proclivity (of which I have uncovered numerous hitherto-neglected examples)[5] for round numbers, sometimes expressing high precision (key rounding here: just 1-part-in-76 million!) in a deceptively round-looking & conveniently compact fashion:

$$S = 223M = 18^K + 10°2/3 = 18^K + 4/135 = 4868^K/270 \quad (6)$$

By contrast to the Aristarchos cycle's smooth dovetailing: had we used Meton's monthlength in eq.5, the remainder would have come out as $10°57'$; computing analogously for Kallippos, it would've been $10°43'$. the Geminos (18:3-6) 19756^d exeligmos produces $10°41'$. The relative ordmag fits of these M to either eq.3 or eq.6 (or eq.5): Meton 10^{-4}, Kallippos 10^{-5}, Geminos 10^{-6}; whereas the agreement of eq.5 (empirical) with eq.6 (Aristarchos) is 10^{-8}. (For Meton's & Kallippos' monthlengths, see Rawlins 1991H fn 1.) NB: Aristarchan eq.6 agrees with later "Babylonian" eq.1 to *1-part-in-24 million*.

A7 Via Censorinus, P.Tannery, & T.Heath, the connexion of Aristarchos to eq.6's quite-particular large number 4868 is neither new nor at all controversial. (See, e.g., Neugebauer 1975 p.603.) The smallest interval containing an integral number of days, Kallippic years, and saros[6] is, by eq.6 (with eq.4), the following, which is the Great Year[7] GY of Aristarchos (as in eq.6, we again drop approximation-signs, in honor of eq.5's near-exactitude):

$$GY = 1778037^d = 270S = 270 \cdot 223M = 60210M = 4868^K \quad (7)$$

Thus, Aristarchos' happy realization that deceptively crude-looking eq.6 was *extremely* accurate (a super-trivial rounding of empirical eq.5 [from eqs.3&4]) made possible his 4868^y Great Year cycle (eq.7): the prime factor 1217 (1/4 of 4868) is embedded right in eq.6.

[4] See Rawlins 1996C eq.8 and attendant discussion. But, ironically, generally-inferior System A's anomalistic monthlength is more accurate. See ‡2 §C3.

[5] See eq.13, ‡2 §E5, ‡3 §E2, Rawlins 1984A n.27, Rawlins 1982G eq.7, Rawlins 2003J fnn 5&38 (esp.), Rawlins 1991W eqs.8&9; and Rawlins 1985H shows Kallippos thought $365^d1/4$ (eq.4) was exactly correct, not rounded.

[6] The plural of saros is often rendered literally as "saroi"; but I think "saros" communicates more unambiguously.

[7] We note in passing that the span of Aristarchos' civil (though not Kallippic or sidereal) GY could have been half of 4868^y (as indicated by Censorinus, P.Tannery, & Heath 1913 p.314f), since halving eq.10 yields: 2434 tropical years = 889011^d.

A8 Once his Great Year was established, he needed to fit the monthlength M into his vast Great Year cycle, and so computed the M implied by eq.7's GY. Suppose he made the clever choice to perform the division of GY by 223 *first*, rounding the result Greek-wise (*nearest timemin*): $7973^d06^h15^m$. (Apt precision: relative accuracy-degradation under 10^{-7}.) In then performing *last* his division of the rounded result by the flagrantly sexagesimalesque number 270, he ensured a neatly-terminating expression for M_A (analog: *DIO 1.2* §R12):

$$M = (1778037^d/223)/270 \approx 7973^d06^h15^m/270 = 29^d31'50''08'''20'''' = M_A \quad (8)$$

[But was eq.1 brother not son to eq.6? Despite fn 2 & p.3 fn 1, we admiringly note Britton's simple alternate (contra-Ptolemy&DR) route via degree-day division (Babylon convention [A's is not known]) from eq.3 or eq.6 to $M = 29^d191°00'49''$ (or $48''$) $\approx 50'' \equiv$ eq.1.]

A9 This (eq.8) is just the eq.1 we wished at the outset to explain. As Aristarchos *could easily post-verify*: this value of cycle-convenient[8] M_A agreed with empirical eq.3 to 1 part in 36 million (an ordmag better than eq.3's accuracy).

A10 Though a sometime critic of Ptolemy, I am here in the happy position of essentially vindicating[9] his *Almajest* 4.2 assertion (repeatedly attacked since Copernicus *loc cit*) that eq.1 came from eq.2. Irony: *Almajest* 4.2 denigrates eq.6, and miscalls it & eq.2 sidereal (as H.Thurston notes), unaware that eq.6, not eq.2, is the more immediate source of eq.1.

A11 Ptolemy's misconception has been lately reborn in the sole plausible-looking point offered against the present paper: the at-first-attractive suggestion that eq.6 is just a rough approximation (merely 1/3 of an exeligmos[10] equation with whole degree remainder: 54 yrs plus 32°). But, even aside from other simple counter-arguments (§A9; fn 2, esp. item [b]), the truth can be fully established by just one single ultra-elementary but central consideration: upon fortuitously-neat eq.6, Aristarchos founded a lunisolar calendaric cycle which was ambitiously designed to extend *for thousands of years*. Why would he do this?[11] —

unless he believed eq.6 to be the most exactly accurate lunisolar equation known.

B Checks from Aristarchos' Metonic "Tropical" Year

B1 Given that Aristarchos' solstice is exactly two Kallippic and eight Metonic cycles (152^y: fn 14) after Meton's famous solstice, we know he accepted the Metonic cycle

$$19^y = 235^u \quad (9)$$

where we use superscript y for "tropical"[12] years. But his new monthlength implied (via eq.9) a "tropical" yearlength differing from those of Meton & Kallippos; so a Great Year containing 4868 Aristarchan "tropical" years equalled (melding eqs.8&9):

$$4868^y = (235/19) \cdot 4868M_A \approx 1778021^d12^h44^m \approx 1778022^d \quad (10)$$

which is 15^d less than the Kallippic-year-based interval of eq.7 in his Great Year scheme — a scheme where everything was arranged to be integral: §A7.

[8] Yes, in eq.8 we are dividing by the very factors (270 & 223) we originally multiplied by, during the development of eqs.5-7. (Note that a paper communicated by A.Aaboe, Swerdlow 1980 [p.292 eqs.a&b], performs the very same kind of move, in order to derive Hipparchos' yearlength from eq.1 via the 76^y Kallippic Cycle.) Between Aristarchos' multiplication and his division, he established his Great Year as his dominant temporal unit (perhaps overoptimistically-intended as an eternal calendaric cycle); and a tiny rounding occurs (similarly at Swerdlow 1980 *loc cit*) when (eq.8) the month is hyperminutely adjusted and re-defined (according to the GY) through the re-division.

[9] This will happen yet again here — see below at ‡3 §§A2&C9.

[10] [Note added 2003. Rawlins 1985K (and the 1st edition of this note) proposed the possibility that some ancient computers might have used 1/45ths of a circle as a basic unit. But A.Jones' recent discoveries (*DIO 11.2* p.30) have severely reduced the acceptability of such speculation.]

[11] See parallel argument at ‡2 fn 25.

[12] For discussion of ancient confusion of Metonic and tropical years, see, e.g., the works of K.Moesgaard (or Rawlins 1996C & Rawlins 1999).

B2 Eq.10 determined Aristarchos' "tropical" year (dovetailed with his Great Year) as:

$$Y_{\mathrm{At}} = 1778022^{\mathrm{d}}/4868 = 365^{\mathrm{d}}1/4 - 15/4868 \tag{11}$$

B3 In continued-fraction format, tropical eq.11 could be written with its distinguishing number rendered sexagesimally:[13]

$$365^{\mathrm{d}} + \cfrac{1}{4 + \cfrac{1}{20 + \cfrac{2}{60}}} \tag{12}$$

B4 Now, it happens that the Vatican holds two rare and precious Greek ancient year-length-lists[14] (Neugebauer 1975 p.601, cited from Vat. gr. 191 fol. 170^{v} & Vat. gr. 381 fol. 163^{v} [see Rawlins 1999 Tables 1&2]), which express various ancient astronomers' yearlengths as continued-fractions. Two listings are given for Aristarchos. One is obviously sidereal[15] and so would not apply here. The other is:

$$\text{Aristarchos of Samos: } [365]\ 1/4\ \kappa'\ \xi\ \beta' = [365]\ 1/4\ 20'\ 60\ 2' \tag{13}$$

B5 The remarkable match of eq.13 to eq.12 provides a gratifying extra reality-check on the foregoing proceedings.

B6 It might be thought that eqs.11&12 do not flow specially from eq.1 because proximate eq.2 would produce the same result. Not so: eq.2 would (if put into eq.10) yield $1778021^{\mathrm{d}}11^{\mathrm{h}}33^{\mathrm{m}}$, thus a remainder (in eq.11) of $-16/4868$, disagreeing[16] with the Vatican ms' Aristarchos data (eq.13): a final, *discrimination-test* reality-check, again revealing Aristarchos' pre-System B possession of the **precise** "Babylonian" month.

[13] According to a common modern style, this would be written 20;02. Britton has suggested interpreting the "tropical" numbers in eq.13 as $365^{\mathrm{d}}1/[4 + 1/(20 + 1/62)] = 365^{\mathrm{d}}1/4 - 31/10052$ instead of eq.12. (Though note: the idea that 60 in eq.13 indicated sixtieths did not originate with DR but with Neugebauer 1975 p.602.) This would destroy the appearance of the Aristarchan number 4868 in eq. 11 and thus imply that eq. 11's display of 4868 is just a spookily spectacular coincidence. But Britton's interpretation would (via eq. 9) actually move M_{A} closer than ever to eq. 1.

[14] These lists were long regarded as mysterious gibberish (Neugebauer 1975 p.602) — until 1980, when long-***JHA***-suppressed Rawlins 1999 treated them (sample analysis at fn 15) as consisting of continued-fraction expressions. (Notably, both mss list Greek yearlength values chronologically-prior to Babylonian values.) [Was the origin of the Greeks' highly useful fascination with cont'd fractions related to ancients' wide but only moderately-utilitarian use of unit fractions?] This analysis revealed both sidereal and tropical years listed under Aristarchos. (Thus, as emphasized in Rawlins 1999, Aristarchos had pre-Hipparchan knowledge of precession: indicating that, reasonably enough, the first public geomobilist was also first to perceive the Earth's precessional wobble.) These induced yearlengths exhibited two characteristic numbers: sidereal, 152 (fn 15); "tropical", 4868 (eqs.11-13), respectively. Both numbers are well-known to be Aristarchan: his solstice was 152^{y} after Meton's (§B1) and his GY was 4868^{y} long. (Note: 4868/32 = 152 1/8 [Rawlins 1999 §B7 bracket].) There are no signs in the Vatican lists, but note that even discarding the final numbers in both Aristarchos entries on the Vatican lists (and using merely the $10'$ [as $-1/10$] and the $20'$ [as $+1/20$]), we get remainders of $1^{\mathrm{d}}/156$ and $-1^{\mathrm{d}}/324$, which are obviously near the ancients' well-known estimates of the excess & deficit of their sidereal & "tropical" yearlengths with respect to the Kallippic $365^{\mathrm{d}}1/4$ calendar. [***Note well*** the many doublings geometrically flowering in the Aristarchos-Hipparchos calendar (fn 17): 1^{d} diff between tropical & Kallippic calendars = $304^{\mathrm{K}}1/4$; saros-cycle return to same longitude = $608^{\mathrm{K}}1/2$; with solar return = 1217^{K}; with lunar return 2434^{K}; with diurnal return = 4868^{K} (eq.7)].

[15] Rawlins 1999 interprets $365^{\mathrm{d}}1/4\ 10'\ 4'$ as: $365^{\mathrm{d}}1/[4 - 1/(10 - 1/4)] = 365^{\mathrm{d}}1/4 + 1/152$. See fn 14 and fn 16.

[16] So, did Hipparchos opt for using eq.2 instead of eq.1? — thus producing his 16-based cycle. (See fn 17.) See another suggestion at Rawlins 1996C §D of his personal contribution [directly attested anyway at *Almajest* 4.2] to the evolution of 345^{y}-cycle-based lunisolar theory. Note: given the chronology-ambiguities implied in ‡3 §D1 below, it is possible that some details of what we have here been attributing to Aristarchos actually had a few debts (positive or negative; examples of latter: Rawlins 1991W §§N7&R12) to Hipparchos' investigations, adjustments, and-or transmissions.

C Comments on the Aristarchan Evidence

Several contemporary scholars have tried to portray Aristarchos' pioneering heliocentrism as a virtually valueless passing-blip on the ancient screen. But the opinions of Archimedes (*op cit*) & Apollonios (*DIO 11.2* ‡4 §L7) show that Aristarchos' bold originality was highly regarded by the brightest ancients. However, a few historians, in anticipation of the current paper (before even seeing its evidence), have suggested there's too little Aristarchan material to make much of. Actually, the ultra-lean evidential situation for Aristarchos only increases the strength of this paper: I can't be suspect of picking among a large Aristarchan corpus to select only the data agreeing with my thesis, since there *is* no such fat corpus to filter. Those who wish to reject the paper's proposals will now have the burden (which I have repeatedly laid on them in other contexts as well — see, e.g., the neat Aristarchos-based Hipparchos-transmitted[17] heliocentrist eqs.23&24 in Rawlins 1991W) of convincingly explaining how such mega-odds fits can meaninglessly emerge from such micro-slim materials.

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[17] There is no question that Hipparchos used Aristarchan data: see *Almajest* 3.1. And the famous $-1^{\mathrm{d}}/300$ remainder in Hipparchos' canonical yearlength is actually a rounding from a cycle of not 300^{y} but 304^{y} (Heath 1913 p.297, Rawlins 1985H, Rawlins 1999 fn 17), perhaps of peculiarly Aristarchan origin by a route revealed in a note appended (at p.42) to *DIO*'s 2001 reprinting (see www.dioi.org) of Rawlins 1999: "The number of years between Meton's famous bedrock −431 Summer Solstice [the epoch of the Metonic calendar] and the Hipparchos [solar] Ultimate-Orbit epoch −127 Autumn Equinox [Rawlins 1991H eq.28: the *only* equinoctial ancient calendaric epoch] is $304^{\mathrm{y}}1/4$. This number is *exactly* one sixteenth the number of years in the 'Great Year' [eq.7] of Aristarchos. That is, the Meton-Hipparchos epoch-interval is $4868^{\mathrm{y}}/16$ — on the nose." (See fn 14.) Now, recall: we found in §B7 that the appearance of 15 instead of 16 in eq.11 was virtually a toss-up; it seems that Hipparchos opted for the 16, thereby adopting a 304 yr calendar that was simultaneously 1/16 of Aristarchos' cycle and [Heath *loc cit*] 16 times Meton's. Note: in the 1 1/2 centuries between Meton's 19 yr cycle and Aristarchos' 4868 yr Great Year, the growth of Greek astronomers' cycles reflected an octodoubling of scientists' temporal vision, up by a factor of *hundreds*; specifically for Meton-to-Aristarchos: about 256 [16-squared or 4-to-the-4th]), paralleling a huge expansion too of man's spatial conception of the universe, also initiated by Aristarchos: §A4, Rawlins 1991P fn 11, & Rawlins 1982G fn 284.