‡4 The instruments used by Hipparchos

by KEITH A.PICKERING

A A revealing gap

A1 More than 2000 years after its compilation, it is now possible to determine with some confidence the kinds of instruments that were used to observe the stars of the Ancient Star Catalog, and in which parts of the sky the various instruments were employed. The fact that there were multiple instruments, and that the ASC was not, as stated by Ptolemy, observed with a single ecliptical astrolabe, does more than provide yet-another proof that the catalog was observed by Hipparchos (for we have more than enough of those already); it also allows us a glimpse into the hitherto unknown workings of astronomy and instrument manufacture as they were practiced at a critical point of ancient Greek science.

A2 The *Almagest* divides the ASC into three sections, for the northern, zodiacal, and southern parts of the sky. Looking at the northern sky, figure 1 plots the absolute errors in longitude for each northern star in the ASC, according to its actual longitude at the epoch of the catalog (which we will take to be -128.0). In looking at the plot, note particularly that there is an odd gap in the plotted stars at about 70° ecliptic longitude. Note also that there is a similar gap at about 250° ecliptic longitude, exactly 180° away. This gap can be more easily seen if we overlay the second half of the longitudes (180-360) on top of the first half, as we have done in figure 2. Note particularly that the longitude errors increase in absolute magnitude as we get close to the gap. For purposes of comparison, figure 3 plots the absolute errors in right ascension by right ascension: the gap disappears.

A3 This gap is significant because it shows us, first, that the northern sky was observed primarily with a single instrument; second, that the instrument was an ecliptic astrolabe; and third, that the astrolabe was of a somewhat different design from the description given by Ptolemy, and indeed different from any previously known to have existed.

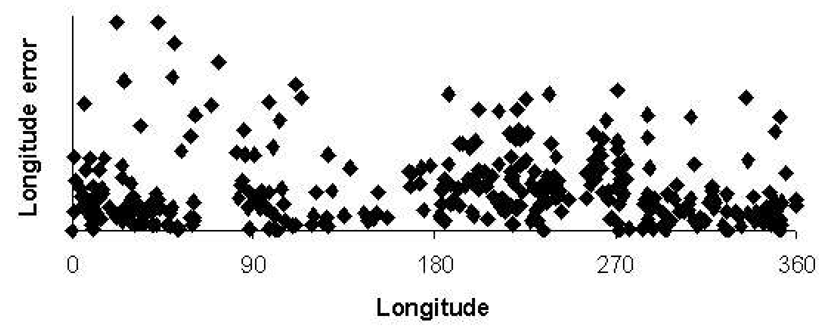

Figure 1: Errors in longitude by longitude, for northern stars in the ASC. Note vertical gaps in the data.

B The astrolabe

B1 The armillary astrolabe used in ancient Greek astronomy is, at first glance, a bewildering maze of nested rings, fitted closely inside each other, that rotate in complex

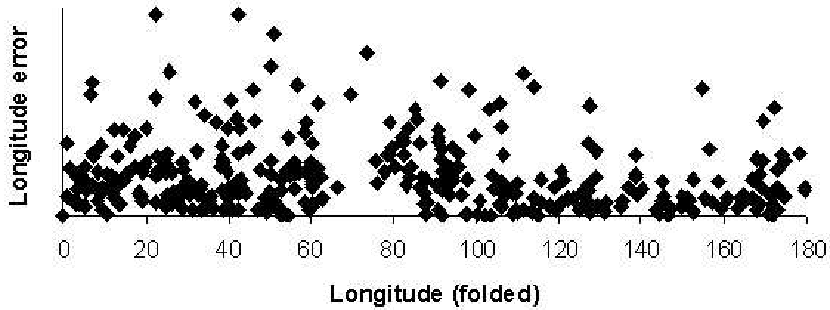

Figure 2: Errors in longitude by longitude, with longitudes overlain. Note vertical gap in the data.

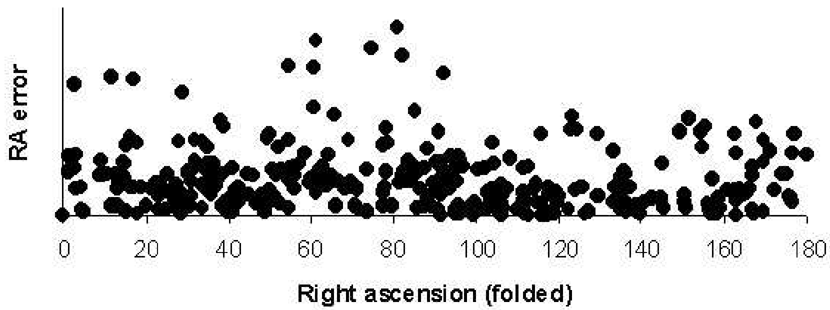

Figure 3: Errors in right ascension by right ascension, for northern stars in the ASC. Note lack of vertical gaps in the data.

ways. Let's look at the way an armillary astrolabe is contructed, from the inside out. The innermost ring (Ring 1) contains a pair of sighting holes or pinnules, diametrically opposite each other, through which the star is sighted. Immediately surrounding Ring 1 is Ring 2, whose inside diameter is fractionally larger than the outside diameter of Ring 1. Ring 1 is constrained so that it rotates inside Ring 2, in the same plane, their edges just touching. Ring 2 has a scale of degrees on its edge, indicating the rotational position of the pinnules on Ring 1. (See figure 4.) If we wished, we could mount Ring 2 on the meridian, and then use the Ring 1&2 assembly as a transit instrument. To do this, we would have to orient Ring 2 so that it points north-south, and so that its zero-degree points on the scale were horizontal, and 90-degree points were vertical.

B2 But to make the Ring 1&2 assembly more useful, we will mount it differently. We construct an outer ring, Ring 3, set vertically so that the whole Ring 1&2 assembly can pivot within it, around a vertical axis. We run axle pins from the 90-degree poles of Ring 2 into the inner edge of Ring 3; so now Ring 1&2 can rotate to any azimuth. To determine the azimuth at which Ring 1&2 is pointing, we add Ring 4, which is fixed horizontally and at right angles to Ring 3. Ring 4 carries another scale of degrees, indicating the rotational position of Ring 2. Rings 3&4 now form a cage, within which Ring 2 rotates freely in

azimuth, while Ring 1 rotates freely in altitude within Ring 2. (See figure 4). The instrument can now be used as a theodolite, since we can determine the altitude and azimuth of any star with it. We will call this arrangement the 4-ring instrument.

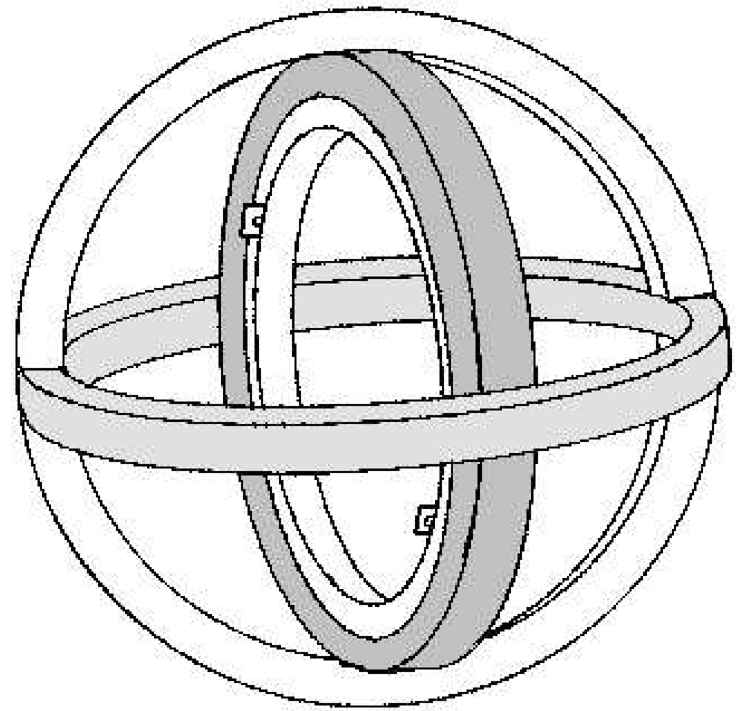

Figure 4: The four-ring instrument. Ring 1 (innermost white) carries pinnules through which the star is sighted. Ring 2 (dark gray) has a scale of degrees. Ring 3 (outer white) holds the polar axis. Ring 4 (light gray) contains the second scale of degrees. When Ring 4 is horizontal, the instrument can be used as a theodolite; when mounted to rotate with the sky, it is an astrolabe.

B3 The 4-ring instrument is capable of pointing to almost any point in the celestial sphere, making it quite useful. In fact, there is only one fly in the ointment to this whole arrangement: at certain rotational positions, Ring 2 becomes so closely aligned with Ring 3 that a star cannot be seen through the pinnules, because Ring 3 gets in the way. There are two such rotational positions, exactly 180° apart. For the same reason, it is impossible to observe very near to the horizon, because Ring 4 gets in the way.[1]

B4 A larger issue with the 4-ring instrument is one of orientation. With Ring 4 oriented horizontally, it makes a fine theodolite, but horizon-based coordinates are of limited utility in astronomy, because the sky moves as the earth rotates. It is much better to mount the

[1] Areas near the axis are also blocked by both Ring 3 and Ring 2.

4-ring instrument so that it rotates too, following the sky.

B5 Recall that the purpose of Ring 3 is entirely structural: it holds the axis around which Rings 1&2 rotate. So the most obvious arrangement is to simply extend that axis, and orient the axis toward the celestial pole. Then the entire 4-ring instrument could be rotated along with the sky. The astrolabe, if mounted this way, would read equatorial coordinates directly, because Ring 4 would be permanently aligned with the celestial equator. All that would be needed would be a way to align the instrument in right ascension.

B6 Although equatorial coordinates are used extensively today, in ancient times ecliptical coordinates were more widely used. So in practice, what was really needed was a way to mount the 4-ring instrument so that: (a) it could rotate with the sky; and (b) Ring 4 would be aligned with the ecliptic instead of the celestial equator. And in the *Almagest* V.1, Ptolemy describes how this was done: a second axis was drilled in Ring 3 (this would have been $23°51'$ from the first). Then the 4-ring instrument was mounted so that the second axis was pointed to the celestial pole. The entire instrument could then rotate (around the polar axis) to follow the sky; while the coordinate readings from Ring 2 and Ring 4 are stuck in a different coordinate frame, tilted in exactly the same manner as the ecliptic is tilted to the equator. And there we have it: the ecliptic armillary astrobale, nearly the same as described by Ptolemy in the *Almagest*.[2]

B7 Except for one big thing. When we drilled the polar axis in Ring 3, at that moment we permanently fixed the ecliptic longitude of Ring 3 along the 90°-270° solstitial colure. This is the great circle in the sky through which both the ecliptic poles and celestial poles run, and now this colure must also run through Ring 3 too, since both instrumental axes run through Ring 3. Now we know that Ring 3 will get in the way of some observations, so if we build an astrolabe this way — as described by Ptolemy — we should expect there to be a gap in observed stars at 90° and 270° ecliptic longitude. As we have seen, there is a longitudinal gap, but it is not at 90°-270°; it is at 70°-250°. This means that the astrolabe which Hipparchos actually used to observe the ASC was built in a somewhat different manner than the one described by Ptolemy in the *Almagest*.

B8 Instead of drilling a second set of axis holes in Ring 3, Hipparchos (or his instrument maker) must have used separate bearing journals to hold the polar axis. There would be two such journals clamped or affixed to opposite sides of Ring 3 at the celestial poles (see figure 5). Since the solstitial colure (which defines 90-270 ecliptic longitude) must contain both axes, the colure no longer contains Ring 3; rather, it is offset by some amount. In the instrument actually used by Hipparchos, this amount was about 20 degrees. This arrangement has a structural advantage, because it avoids putting another set of holes in Ring 3, which has already been weakened by the holes for the ecliptic axis.

B9 If he had used more than one astrolabe for observing the Northern sky, Hipparchos could have arranged to have the journals on astrolabe #2 mounted on the opposite sides of Ring 3 than the arrangement on astrolabe #1; so the blind spot of astrolabe #2 would be at 110-290, and the blind spot of one instrument could be covered by the other. Therefore it is apparent that large parts of the northern sky were observed with a single instrument — or nearly so.

B10 You recall that there is another blind spot, along Ring 4. This ring falls right on the ecliptic, so we might expect to see a gap in the data here, too, just as we found in the longitudes. In the northern sky, only one constellation (Ophiuchus) dips all the way down to the ecliptic; but there is no such gap along the ecliptic in Ophiuchus. In fact, there is no such gap among the stars of the zodiacal constellations, either.

[2] For purposes of simplicity, we have left out Ring 5, which is used only as an aid to orientation. Ring 6, which is fixed to the earth as a strutctural support for the whole instrument, holds the second axis, which points toward the North Celestial Pole.

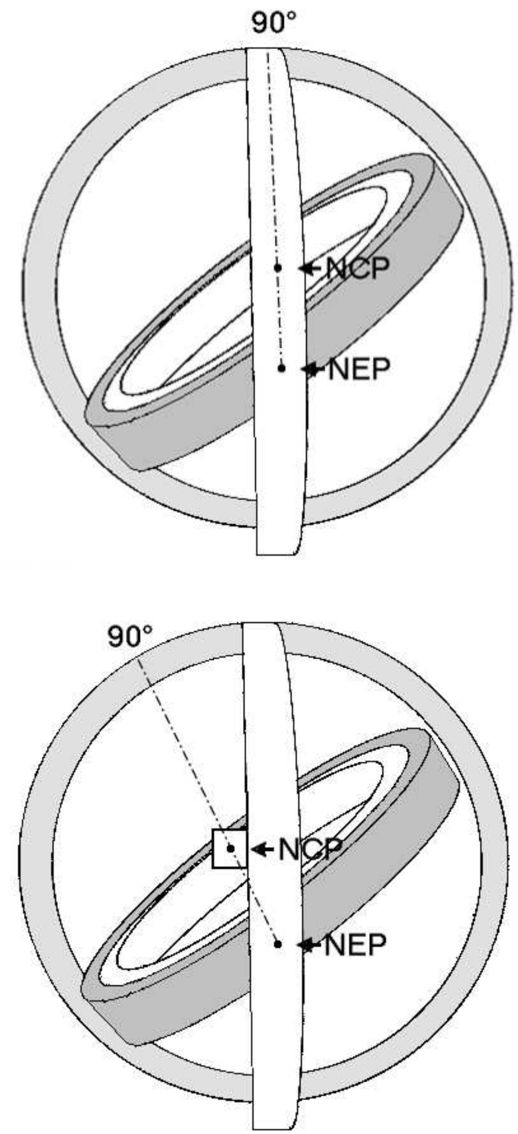

Figure 5: The four-ring instrument viewed from above the poles. Ring 2 (dark gray) pivots around the North Ecliptic Pole (NEP), while the entire 4-ring instrument rotates around the North Celestial Pole (NCP) to follow the sky. (The NCP axis is affixed to an outer Ring 6, which is not shown in the diagram.) According to the *Almagest*, the NCP axis is drilled in Ring 3 (top); in the astrolabe used by Hipparchos, the NCP axis is carried on a separate bearing journal (bottom). The 90-270 solstitial colure is the great circle joining the two axes — along Ring 3 (top) or offset from it (bottom).

B11 So there must have been a different instrument or a different technique (or both) for observing right at the ecliptic. One possibility is a second set of pinnules.[3] The primary pinnules would be mounted on Ring 1 at diametrically opposite positions, as already described; while the second set would be mounted above these, a little more than one ring-width away. Thus, the sightline through the first set would be exactly parallel to the sightline through the second set. When the first set was too close to the ecliptic to observe, the second set would still be able to see over the top of Ring 4.

B12 I have been unable to find similar gaps in either the Zodiac or the South sections of the ASC. This implies that the instrument used in the North was different than the one(s) used in other parts of the sky.

C Gap Characteristics

C1 Are there bright stars in the gap that Hipparchos usually would have taken, or is the reason for the lack of cataloged stars simply that there are no bright stars in this region of the sky? In other words, is the gap real? As it turns out, there are only five stars in the Northern sky brighter than magnitude 3.9 that Hipparchos left out of the catalog: χ UMa, α Lac, 46 LMi, 109 Her, and α Sct. Two of these five (109 Her and α Sct) are in the gap. Since the gap represents only about 5% of the sky, this is clearly a significant number.

C2 The gap is caused by the physical presence of Ring 3, which has a constant physical width. But the longitudinal width of Ring 3 increases toward the ecliptic pole, because the lines of longitude converge there. We can determine the relative thickness of Ring 3 by close examination of the edges of the gap. In figure 6, I have plotted the region near the gap in latitude and "folded" longitude, along with lines indicating the position that the gap would have if Ring 3 was centered at 69.5 - 249.5 and had a width of 3.7 degrees. These parameters fit the actual gap quite well (although smaller widths cannot be excluded).

C3 Similarly, in figure 7, I have plotted all stars of magnitude 4.5 or brighter that are missing from the catalog, with the same gap limits. Note particularly that there are no missing stars this bright above latitude 75. This is a good indication of the polar limits of the astrolabe, and shows the region in which a different instrument was probably used. This also explains why there are a couple of holdout stars present in the gap: the holdouts are both at very high latitudes.

C4 The edges of the gap are between 4 and 5 degrees apart at the ecliptic. The exact edges depend on how far north one chooses to assume was observed with this single instrument. The gap is actually two adjacent gaps: one in which the lower pinnule is blocked by Ring 3, and one in which the upper pinnule is blocked. Therefore, the 5-degree width of the gap implies that the physical width of Ring 3 was between 2 and $2^\circ.5$ degrees. The exact center of the gap is a bit tricky to pin down, but it seems to be very close to $69^\circ.5$ of ecliptic longitude.

C5 Further, between these two adjacent gaps, at the very center, there is a very narrow "gap within the gap," where a star lying at that precise longitude should be visible. This is because, when Ring 2 is *exactly* aligned with Ring 3, neither pinnule on Ring 1 is blocked by Ring 3: the line of sight passes along the edge of Ring 3 just as if it were a wide extension of Ring 2. As it turns out, there is in fact a cataloged star lying almost exactly at the center of the gap: ϵ UMi. But since it also lies at a very high latitude, there is no guarantee that it was observed through the "gap within the gap," rather than in the same manner as other stars near the ecliptic pole.

C6 A ring about 2 degrees wide is rather narrow, structurally speaking, which in turn places limits on the material used to construct the astrolabe. For example, if Ring 3 was 50 cm in diameter, it could be no more than about 1 cm (perhaps less) in width. I tend to

[3] Ptolemy does not mention a second set of pinnules in the *Almagest*, but given the foregoing, he cannot be taken as a wholly reliable source on the construction of astrolabes.

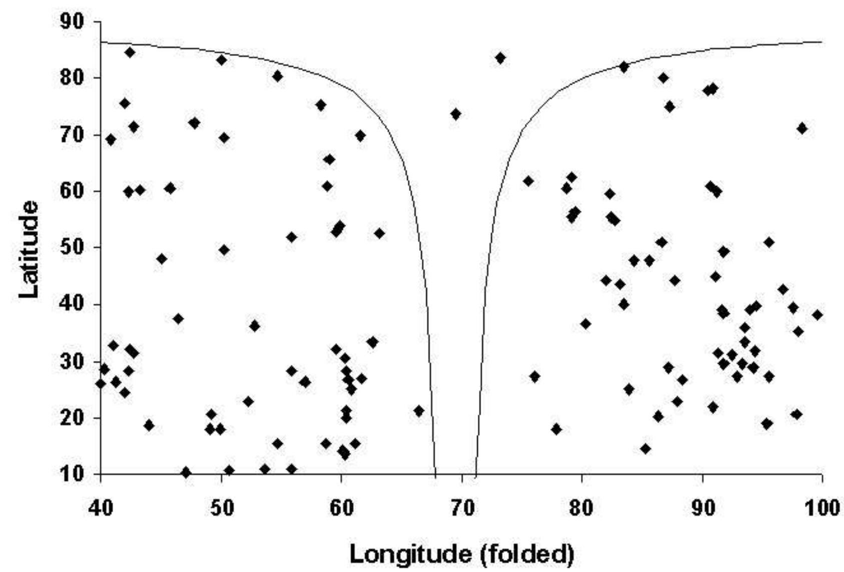

Figure 6: Close view of the gap in cataloged stars. The lines show the limits of a gap 3.7 degrees wide centered at 69.5 - 249.5 degrees longitude.

doubt that a wooden instrument of this narrow aspect ratio could be stiff enough against the weight it must support to be very accurate; bronze seems a more likely material.

D Epoch of the Northern Catalog

D1 The single-instrument hypothesis implies that the northern sky was observed all at once, before the instrument had time to become worn or damaged; in other words, a matter of months or a few years, rather than decades. Careful analysis will allow us to determine the epoch of this northern observational effort.

D2 After subtracting Ptolemy's $2^\circ\ ^2/_3$ false precessional constant, we can reconstruct the actual longitudes of these stars as observed by Hipparchos. Due to precession, stars advance from west to east parallel to the ecliptic, maintaining their same ecliptic latitudes, but increasing their ecliptic longitudes at a rate of about $83'$ per century. So, as a first cut, we can simply take these reconstructed Hipparchan longitudes and assume that they were (on average) correct as measured, then find the epoch at which such an assumption would be true. For the northern stars, this works out to -157 ± 59 years.

D3 There is a problem with this procedure, however, because the longitudes observed by Hipparchos were not actually correct, on average. There is a systematic error which we must account for. The longitude of the stars is determined ultimately by reference to the Sun. The Sun is observed just before sunset, on a day just after new Moon. The longitude of the Sun is known from theory, and the difference between the Sun and Moon gives the Moon's longitude; then, after sunset of the same day, the difference between the Moon and a fundamental star is observed, to give the longitude of the fundamental star; and finally, the longitudes of individual stars are observed by their difference from the fundamental star. But each of these steps requires the astrolabe to be briefly clamped in position while the

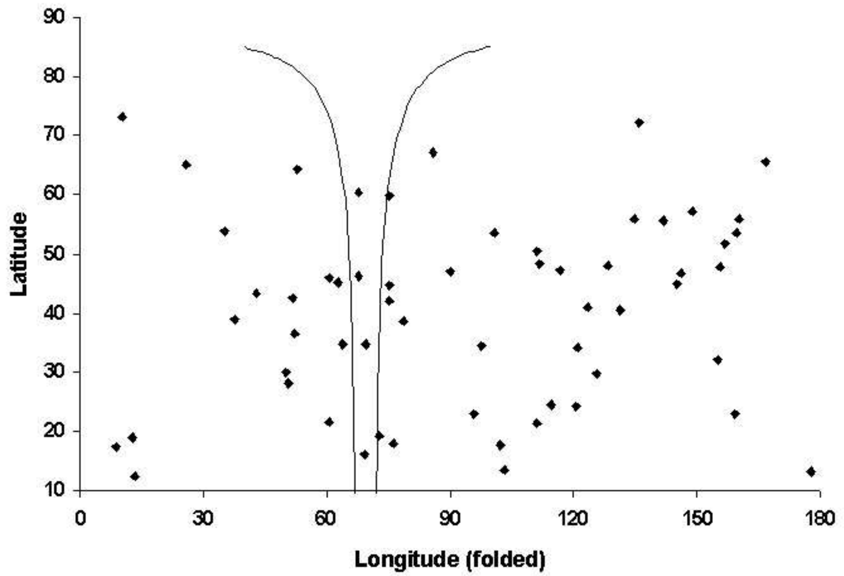

Figure 7: Stars brighter than magnitude 4.5 missing from the northern part of the ASC. Note that none are missing above latitude 75°, possibly indicating that a different kind of instrument was used in this small region of the sky.

measurements are being made; and these successive clampings tend to push the longitudes lower than true, because the earth rotates during these brief intervals. In other words, there is a systematic error in rotation of the astrolabe around the equatorial axis.

D4 Rawlins 1982 has shown that misrotation of the astrolabe with respect to the real sky will make itself known by the presence of a cosine error wave in the observed latitudes. Further, the amplitude of this cosine error wave is proportional to the amount of astrolabe misrotation. And in fact there is just such an error in the latitudes of the northern stars. This error wave has an amplitude of 10.6 $\pm$1.8 arcmin, implying that the astrolabe was systematically misrotated by 24.2 $\pm$4.2 arcmin. It took precession 29.2 years to move a star that far in longitude, meaning that the actual epoch of observation for the northern stars was -128 $\pm$59 years. This is very nearly the epoch implied by Ptolemy's precessional constant.

References

Rawlins, Dennis (1982). An Investigation of the Ancient Star Catalog. *PASP* 94, 359.

Toomer, G.J. (1998). *Ptolemy's Almagest*. Princeton University Press.