‡1 On the Orientation of Early Egyptian Pyramids

by Hugh Thurston[1]

A Introduction

A1 Eight early Egyptian pyramids are oriented amazingly accurately; their eastern and western sides are less than degree from a true north-to-south line (§A3). If we arrange them in order of date, we find that with two exceptions their orientations drift slowly but steadily clockwise.
A2 This suggests that the pyramids were aligned by a method vulnerable to precession. The two exceptions, the pyramids of Khafre and Sahure, are out by the amount expected[2] but in the opposite direction, suggesting that the method was reversible[3] and was used in reverse for these two pyramids.
A3 If we reverse the sign of the deviation for the two exceptions cited in §A2 (clockwise deviations here are positive), we have the following data, listing pyramids in chronological order:

Site	Pharaoh	east side	west side
Meidum	Sneferu	$-20'.6$	$-18'.1$
Dahshur, south	Sneferu	$-17'.3$	$-11'.8$
Dahshur, north	Sneferu	$-\ 8'.7$	
Giza	Khufu	$-\ 3'.4$	$-\ 2'.8$
Giza	Khafre	$+\ 6'.0$	$+\ 6'.0$
Giza	Menkaure	$+12'.4$	
Abusir	Sahure	$+23'$	
Abusir	Neferirkare	$+30'$	

The deviations of the east and west sides are quoted from a paper [1] by Kate Spence, who used a number of sources, primarily the work of J.Dorner — especially his 1981 dissertation [2].

B Suggested Methods: Haack

B1 Steven Haack [3] suggested that the Egyptians searched for a star that appeared to rise precisely due east and then aligned their pyramids' north and south sides on it.

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[2] In spite of alternate possibilities, etc, noted elsewhere here (p.2 fn 2 and p.3 fn 2), DR must in fairness take space to emphasize the prime strength of Spence's theory (& Haack's earlier one): for her eight-pyramid sample (where she has dropped one of Haack's data [Zoser]; added another [Dahshur-Red]; & altered others [Sahure & Dahshur-Bent], presumably for the better), the temporal trend of misorientations' absolute magnitudes is monotonic (in either direction from her theory's null-error time) — which is consistent with the precession theory's explanation. (Though, some of the standard deviations estimated by Spence [2000 Table 1] are comparable to [and in some cases exceed] the associated differences which establish this monotonicity.)

[3] See p.2 fn 2.

B2 The azimuth where the star appeared to rise would rotate clockwise. The azimuth where it appeared to set would rotate in the opposite direction at the same rate, and Haack suggested using this to orient Khafre's pyramid.
B3 However, the Egyptians cannot have done this.[4] As I saw on a recent visit (having been forewarned by Kate Spence), the plateau at Giza slopes upward to the west and when the Egyptians levelled the ground for Khafre's pyramid they left a cliff some 10 metres high a little way to the west. This would form a high local horizon and cause a star that rises due east to set much too far south (of due west) to account for the slight deviation of the pyramid.
B4 Haack suggested that the ancient surveyors used the star α Arietis for the two pyramids at Abusir, β Scorpii for the others.[5]

C Suggested Methods: Spence

C1 Kate Spence suggested that the Egyptians thought that the pole was directly between the stars Mizar and Kochab, and indeed in 2467 BC it was [1]. For Mizar above Kochab, the azimuth would drift clockwise, so most pyramids were aligned with Mizar above Kochab. For the two exceptions, Kochab was above Mizar. Both drifts were at $0'.31$ per year [4].
C2 2467 BC is very late in, or later than, the dates given by various authorities for the fourth dynasty. Dates for the start of the dynasty vary from 2640 BC (quoted in D.Arnold, *Building in Egypt* [1991] as from "R.Krauss, 1985") to 2575 BC (Baines & Malek, [5]). Dates for the end of the dynasty vary from 2504 BC (the earliest accepted by Beckerath [6]) to 2454 BC (the latest accepted by him).

D Suggested Methods: R&P

D1 Dennis Rawlins and Keith Pickering suggested [4] that the Egyptians thought that the pole was equally far from Thuban and 10i Draconis,[6] and indeed in 2627 BC it was. This would imply that when these stars were aligned horizontally, the point midway between them was due north.
D2 Most pyramids would be aligned with the two stars above the pole; the two exceptions, under the pole. This alignment would drift at $0'.274$ per year. The suggested date 2627 BC is very early in, or earlier than, the dates for the fourth dynasty given in §C2.

E Observations

E1 According to Haack's method, the Egyptians simply watched[7] for a star that rose due east or set due west and they aligned each pyramid on the rising or setting of this star. The observations had to be made at the time of year when the star rose or set at night.

[4] See [4]'s last paragraph.

[5] These stars had two different speeds in azimuth: $0'.34$ per year for α Arietis, $-0'.39$ per year for β Scorpii. Both values for azimuthal speed provided here are for 2600 BC, when α was 332° for Hamal (α Ari) and 180° for Acrab (β Sco). (During the period of our interest, Hamal's speed increased merely $0'.004$/yr/cy, while Acrab's was virtually constant.) In general: $dA/dt = p \sin\epsilon \cos\alpha \sec\gamma \csc A$ (on horizon), which equals $0'.39\cos\alpha$ for due E or W at geographical latitude $\gamma = 30°$ and (for mid-3rd millennium BC) annual precession $p = 0'.82$ & obliquity $\epsilon = 24°$. If dA/dt is positive the rising and setting points rotate northward; otherwise, southward.

[6] The star 10i Dra is of variable magnitude (4 1/2-to-5). In almost exactly 2800 BC, adjacent Thuban became the most proximate pole star in history (brighter than 4th magnitude): less than $0°.1$ from the exact pole. But, starting in 2627 BC, no brighter star was nearer the pole than 10i Dra, for the next 11 centuries. (And no brighter-than-5th-magn star nearer for the next 9.)

[7] A direct method would be: just looking for stars that appeared to set 180° from where they rose.

E2 For the other two methods the Egyptians needed to find the pole: not only its direction but also its altitude (which is 29°58′-59′N true[8] at Giza, a little less at the other sites). If they didn't know where the pole was, they would not know that it was collinear with Mizar and Kochab or equidistant from Thuban and 10i Draconis. These two methods are most easily carried out by using a plumb-line. Stand at a corner[9] of the pyramid-to-be and watch the two chosen stars. When they are aligned, vertically or horizontally as the case may be, they are north. Set up a plumb-line north of the position of observation, high enough to cover the stars and very nearly touching the ground.
E3 A little earlier the next night, go to the same spot, taking with you a small ring fixed to the top of a tripod formed by three sticks lashed together. (If you want to test the practicability of this for yourself, you will find that a camera tripod serves very well.)
E4 For the vertical alignment method, watch one of the stars through the ring, moving the tripod to a position where the plumb-line covers the star. Move the tripod sideways to keep the star covered (the movement needed will be quite slow) and when the plumb-line covers the other star, drop a plumb-line from the ring. The line between the two plumb-lines gives the orientation sought.
E5 The horizontal alignment method is similar. Keep the plumb-line covering the midpoint between the two stars and drop the plumb-line when they are judged to be horizontally aligned. Because the two stars are close together (merely 1°1/2 apart), a midpoint judged by eye will not be far out, and because the midpoint itself is so near the pole (barely 3/5 of a degree away), a slight misjudgement in the time of horizontal alignment will not move the midpoint very far sideways.

F Precedents

There is precedent for using two stars collinear with the pole to find north: today we (or at least Boy Scouts & Girl Guides) use the Pointers. And there is a precedent for using a vertical alignment: Polynesian and Micronesian navigators knew that when the Southern Cross was upright it was pretty well due south.[10] There is no known precedent for either of the other methods.

[8] The apparent (not true) altitude of the pole is 30°00′ (1/12 of a circle) as seen from all three Giza pyramids. (Latitudes: Khufu 29°58′.7 N, Khafre 29°58′.5 N, Menkaure 29°58′.3 N.) Whether this is accidental is discussed in D.Rawlins, "Ancient Geodesy: Achievement & Corruption" (*Vistas in Astronomy 28*:255-268 [1985] pp.255-256).

[9] See Mark Lehner *The Complete Pyramids* London 1997 pp.212-213, 220 (& p.109).

[10] If based upon verticality of Crux's mast (α&γ Cru), the Pacific peoples' S.Cross-method would have provided exact south just before 1000 BC; or just after 3000 BC, if horizontality of the crossbar (β&δ Cru) was used instead. The UMa Pointers' line hasn't pierced the pole since the mid-13th century AD, but the Pointers are still popular anyway — primarily for finding Polaris (α UMi), a star which provided north by being about 1° from the true pole, as α Dra & 10i Dra were, back in the 27th century BC. (The gyroscopically-precessing celestial pole was 71′ from Polaris in 1908 [when the Scouts were founded]; now, 43′ away; in 2102, the pole will pass within 28′ of Polaris and then recede from it.)

G The Intervals of Time Between the Pyramids

G1 Except for Sneferu's 2nd & 3rd pyramids, the dates at which the pyramids were started will be (fn 12) within a year or two of the lengths of the pharaohs' reigns; a pharaoh will obviously start his pyramid early in his reign.
G2 The lengths of the reigns are fundamental to Egyptian chronology. Early chronology depends on a Sothic date in the 12th dynasty, which pins the 7th year of Senosret III at 1872 BC. (Most Egyptologists regard Sothic dating as valid. Those who don't have no anchor point.)
G3 Dates before Senosret III depend entirely on estimating the lengths of individual reigns and working back step by step.
G4 The fourth and fifth dynasties, together with the sixth constitute the Old Kingdom. Then came a period of chaos known as the first intermediate period (FIP), after which the eleventh dynasty started the Middle Kingdom.
G5 Our main source for the lengths of the reigns is the Turin canon, a papyrus document which unfortunately is far from complete. It can be supplemented by a late compilation in Greek by Manetho, by the Palermo stone, and by inscriptions at Abydos & Saqqara.
G6 We can put a lower limit on the length of a reign if we find a record of an event such as the 24th cattle-count in the reign of Sneferu. Cattle-counts probably took place every 2 years; if so, Sneferu must have reigned for at least 48 years. (See R.Stadelmann, "Beiträge zur Geschichte des Alten Reiches," *MDAIK 43*:229-240 [1986].)
G7 The records are complete enough to take us back from Senosret III to the beginning of his dynasty, the twelfth (Baines & Malek describe this date as "known with precision", [5], page 36), and with fair confidence to the beginning of the eleventh.
G8 We then have to deal with the FIP, about which there is practically no information. Cyril Aldred [8] makes it 149 years. Beckerath gives it a maximum of 50 years and a minimum of zero [6].
G9 Obviously for the Old Kingdom we should not put much faith in absolute dates, However, we can look at the lengths of the reigns. In the Turin canon, most of the names of the pharaohs are missing; but all except two can be supplied from tablets at Saqqara and Abydos. The two in parentheses below are from Manetho: we do not know the Egyptian forms of these names. This gives the table below.

Pharaoh	Length of Reign
Sneferu	24^y
Khufu	23^y
Djedefre	8^y
Khafre	$2[]^y$
(Bicheris)	$[]^y$
Menkaure	$[]8^y$
Shepseskaf	4^y
(Thamphthis)	2^y

G10 Manetho had two pharaohs named Suphis (presumably Khufu and Khafre) with reigns of 63 and 66 years. Beckerath assumed that he obtained the 63 by adding 40 to the correct number, to agree with a remark by Herodotus that the pharaohs who built the enormous pyramids reigned over 60 years. Beckerath assumed that Manetho did the same for Khafre, and that therefore his missing digit is 6. He altered Sneferu's reign to 35 years. (We saw in §G6 that this is still too low.) At one point Beckerath stated that Menkaure's reign was 18, 28, or 38 years, presumably because there was not room on the missing fragment for more than three of the hieroglyphs for 10. He settled for 28 in his final list.

He allotted 7 years to Bicheris from the 7 allotted to Sebercheres by Manetho (who allotted 22 to Bicheris). He altered Djedefre to 9 and Shepseskaf to 5.

G11 This, together with Beckerath's dates for the fifth dynasty gives column Bk in the table below. For comparison, columns B&M and CAH give the corresponding intervals from Baines & Malek [5] and the Cambridge Ancient History [9], respectively. Columns Sp and R&P give the time in years to produce the changes in orientation of the eastern sides of successive pyramids, as shown in §A3, using the methods of Spence (Sp) and Rawlins & Pickering (R&P), respectively.

	Bk	B&M	CAH	Sp	R&P
Sneferu to Khufu	35^{y}	24^{y}	24^{y}	55^{y}	63^{y}
Khufu to Khafre	32^{y}	31^{y}	31^{y}	30^{y}	34^{y}
Khafre to Menkaure	33^{y}	30^{y}	25^{y}+	21^{y}	23^{y}
Menkaure to Sahure	43^{y}	32^{y}	41^{y}	34^{y}	39^{y}
Sahure to Neferirk.	13^{y}	12^{y}	14^{y}	22^{y}	25^{y}

When reading this table bear in mind that the best historical estimate for Sneferu-to-Khufu is none of those listed. It is 48 (§G6).

G12 Spence's method[11] makes Khufu's reign start about 2480 BC.[12] R&P make it about 2638 BC. Haack made the fourth dynasty start about 2640 BC.

G13 I don't know what chronologists will make of this. So far they seem to have ignored it. But at least it confirms that they were right to reject Manetho's long reigns for the pharaohs who built the Giza pyramids and backs Stadelmann's case [cited §G6] that estimates for the length of Sneferu's reign cited in §G11 are substantially too short.

H Appendix: Some Mathematics

H1 Spence's paper [1] gave rise to some trigonometry which, though not relevant to Egyptology, is of interest in itself. And it sparked the interest of Rawlins & Pickering.

H2 Let us denote by ϕ the angular distance between the celestial pole (as seen from Giza) and the plane through Giza, Mizar, & Kochab at instants when these stars are aligned vertically. Spence interpreted ϕ as the deviation from north given by the vertical line. It is not. (As pointed out by Rawlins & Pickering in [4] and agreed to in Spence's reply.)

H3 Figure 1 shows the situation. G is Giza. GP is in the direction of the pole. GN is horizontal, and PN is vertical, so GN points horizontally north.

H4 How do we find the angle between GP and the vertical plane through Giza and the stars? Answer: drop a perpendicular PQ to this plane; then the angle PGQ is the angle required (namely ϕ).

[11] The misorientations of Khufu's sides (Dorner 1981 p.77), all W: N $2'28''$, S $2'31''$, W $2'47''$, E $3'26''$. (Note: parallel Khufu W&E sides' azimuths would disagree by $4''$ from Earth-sphericity; not negligible if we display $0'.1$ precision.) Taking the W side (above: p.3) as the closest approximation to the ancient surveyor's original orientation-error, R&P's Thuban-10i Dra method produces 2636 BC for the Khufu pyramid's start. [Proceeding as in fn 12: $-2626 - 2'47''/0'.274 = -2636 = 2637$ BC.] The R&P dates for Khufu are nearer conventional ones than Spence's (as noted in [4]).

[12] Taking the mean $3'.1$ W misorientation (fn 11 or §A3) of the Great Pyramid's E&W sides, one can divide by Spence's original (flawed: §H2) Mizar-Kochab $0'.28$ speed ([1] Fig.4 line a's slope) and subtract this from -2466 (2467 BC), the date of null error (when the pole was exactly on the Mizar-Kochab line): $-2466 - 3'.1/0'.28 = -2477 = 2478$ BC. She further corrected for a presumed 2^{y} gap (§G1) between the pharaoh's ascension and his pyramid's start, to find 2480 BC for the ascension date. (Repeating her calculation with correct slope [$0'.31$/yr] yields 2479 BC, instead; it's a tiny difference in this case, but date-corrections for some of the other pyramids are ordmag 10^{y}.)

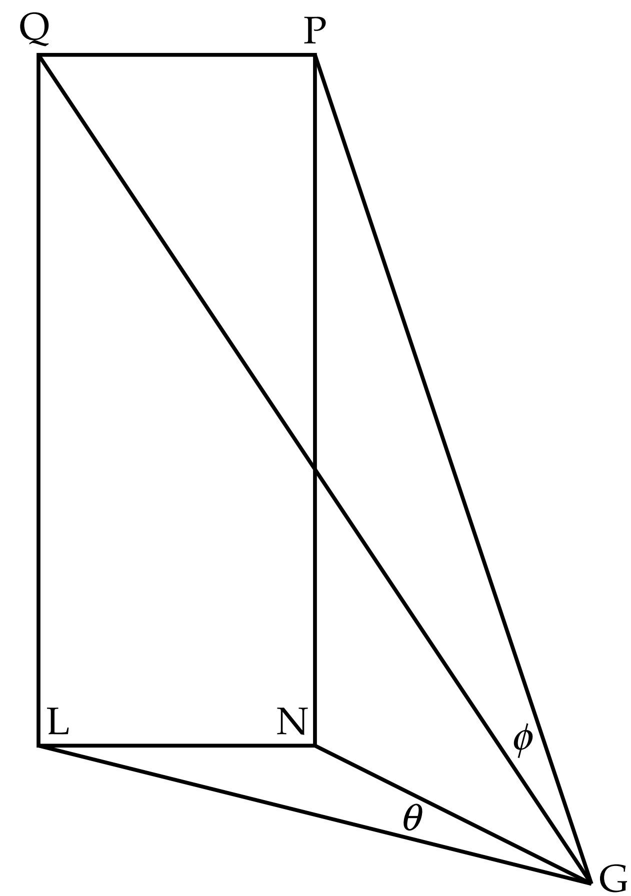

Figure 1: Showing the relation between θ and ϕ. [See §H3.]

H5 Drop a perpendicular QL to the horizontal plane through Giza. Then GL is the ground-level orientation given by the stars. The angle LGN is the deviation from north; let us call it θ.

H6 Because PQ = LN while PG and QG are greater than NG and LG, the angle PGQ is clearly smaller than the angle NGL. That is to say, $\phi < \theta$. By elementary trigonometry, we can find the exact relation between θ and ϕ. The angle PGN is the altitude of the pole, which is Giza's latitude γ. Then (in Figure 1):

$$\sin\phi = \frac{QP}{GP} = \frac{LN}{GP} = \frac{LN}{GN} \cdot \frac{GN}{GP} = \sin\theta\cos\gamma \tag{1}$$

H7 An indirect calculation can be done on the celestial sphere (e.g., Figure 2), which is a mathematical fiction devised by the ancient Greeks who had no device like the vector for dealing with directions. It is a large imaginary sphere with its centre at the centre of the Earth. Any direction in space is represented by the point on the sphere in that direction, and angles are represented by arcs of great circles on the sphere. The Greeks could then use spherical trigonometry in their calculations.

H8 Figure 2 illustrates the celestial sphere centred at Giza. G is Giza. GZ is vertically upwards. P is the celestial north pole. K and M are Kochab and Mizar when they are aligned vertically, so the great circle through them goes through Z. Both L and N are on the horizontal plane through Giza. PQ is an arc of the great circle through P perpendicular to plane ZMKL. Then ϕ is represented by the arc PQ and this in fact is how Spence quoted it. θ is the angle LZN.

H9 If we denote the interior angles of the spherical triangle ZQP by Z, Q, and P, and the sides by z, q, and p, we have (by the law of sines): $\sin z/\sin Z = \sin q/\sin Q$. But here $Z = \theta$, $z = \phi$, $Q = 90°$, and $q = 90° - \gamma$. So we have:

$$\sin\phi = \sin\theta\cos\gamma \tag{2}$$

just as in eq.1.

H10 Since ϕ and θ are tiny, the horizontal arc ϕ' from P to ZMKL is very close to the arc PQ.

$$\phi' = \theta\cos\gamma \tag{3}$$

Rawlins & Pickering used ϕ' instead of ϕ. The difference, if θ and ϕ are small, is negligible. (In case you are interested, if $\gamma = 29°59'$ and $\theta = 12'$, then the exact formula eq.1 gives $\phi = 0°.1732341$, while the simpler relation eq.3 gives $\phi' = 0°.1732342$.)

H11 As a result, the intervals between pyramids used by Spence [1] have to be divided by $\sec\gamma$ (which varies from 1.155 at Giza to 1.149 at Meidum).

References

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[7] Thurston, Hugh, *Early Astronomy*, 1994.
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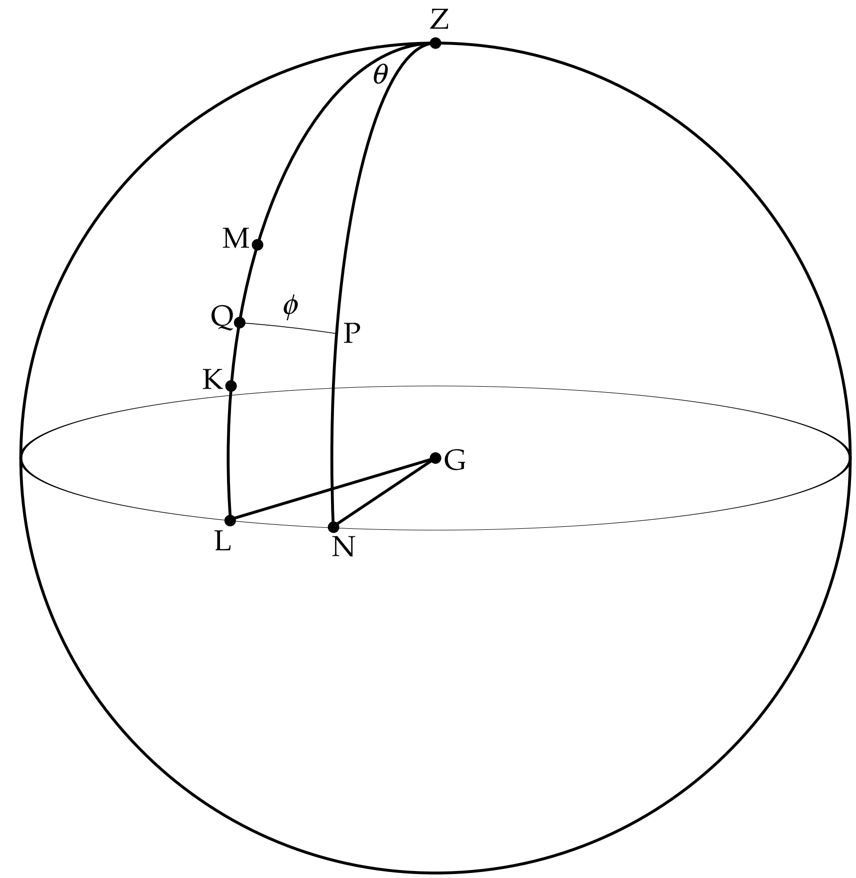

Figure 2: The relation between θ and ϕ using the celestial sphere. [See §§H7&H8.]