‡2 Pytheas' Solstice Observation Locates Him: Cape Croisette

Pytheas' Solstice: Oldest Vertical-Instrument Transit Observation
Why Has No Historian Taken Pytheas' Precision Seriously?
Or Bothered Consulting a Map of Marseilles?

Summary

The earliest person known as a scientist-explorer is Pytheas, native & citizen of the Hellenistic colony of Massalia: modernly Marseilles, still the main city of south-coastal France. A legendary figure, Pytheas was known (‡3 §G1) as an able mathematician, astronomer, and geographer. In the history of the exact sciences he is primarily remembered for his Summer Solstice observation (‡3 eq.10) of the shadow/gnomon ratio at Massalia at Local Apparent Noon:

$$s_{\rm S}/g = \frac{41\frac{4}{5}}{120} \doteq \tan 19^\circ 12' \tag{1}$$

It is the purpose of the present article to establish several points.
[1] The reality & historical utility of eq.1 is shown by two independent indicia:
[a] The ordmag $1'$ precision of his Marseilles datum is that expected of real outdoor pre-telescopic measurement.
[b] Said precision narrows Pytheas' location to a coast near Marseilles (Fig.1) which turns out to be the ideal Marseilles-region location for an astronomical observatory — far better than Marseilles proper.
[2] Pytheas' Summer Solstice observation was presumably based upon the average of repeated sightings (perhaps in annual bunches) at his long-term home-town observatory, which would yield a precise result constituting *the oldest extant raw astronomical transit observation*.[1]
[3] The exact location of his observatory is recoverable to a precision of ordmag 1 mile — in both latitude and longitude — at Cape Croisette (a few miles south of Marseilles), a vantage-point *having an astronomer's ideal southern view over the Mediterranean*.

A Having a Fortuitous Ball

A1 We have elsewhere (e.g., ‡1 fn 15 & ‡3) dispensed with a 2002 Muffia-descended last-gasp attack upon one of the glories of rational scientific history — specifically: upon Aubrey Diller's immortal priority in proving Hipparchos' use of spherical trig and an accurate obliquity in the 2nd century BC. But we happily have a positive outcome from the Muffia's 75y "hubbub" on the Diller issue (to borrow MuJHA p.15's flip sneer at the firmness of Diller-DR's diamond-clear discovery): we will respond to the offending paper's mis-adducement of the famous S.Solstice gnomon observation of Pytheas of Marseilles (which alleges it was just a calculated non-observation), by running with the ball fortuitously lobbed our way, recognizing the datum as that of a patently high-precision ***observation*** — and thereby locating the Mediterranean spot near Marseilles where this legendary astronomer-navigator-explorer did his astronomy: Cape Croisette (Fig.2), $0^\circ.1$ south of Marseilles-harbor proper (Fig.1).

[1] Without certainty, one presumes Pytheas observed before Timocharis since the latter probably used a transit circle, an advance over the gnomon. Anyway, Timocharis' star declinations are not raw data.

A2 MuJHA p.17 having claimed that the Summer Solstice datum (eq.1 or ‡3 eq.10) of Pytheas was not an observation, we explore (as scientists should) an alternate possibility, namely, that Pytheas' eq.1 was a real gnomon observation. (Which is actually, *a priori*, much more than a possibility.) We know that many Greeks' gnomons were vertical & *asymmetric*. (See, e.g., diagrammed discussions at Manitius 1912-3 1:419-420 & R.Newton 1977 pp.38-39. Also developments in, e.g., Rawlins 1982G & Rawlins 1985G pp.260f.) This produces a shadow corresponding (eq.5) to the S.Solst zenith distance Z_s of the *top* (not center) of the solar disk: the upper limb. (I.e., measured Z will be $16'$ [the solar semi-diameter ssd] less than the Z of the solar center, a fact many well-known Greeks were naïve about.)[2] Thus, a solstitial s_s/g with such an instrument will produce a latitude L which is $16'$ less[3] than the true value. A useful 1st estimate of the uncertainty in Pytheas' Z_s follows from checking its limits (via eq.1), knowing ancient rounding practices (discussed at, e.g., Rawlins 1994L §B3), which used degree halves, thirds, fourths, fifths, & sixths:

$$s_s/g = \frac{41\frac{3}{4}}{120} \doteq \tan 19°11' \qquad \& \qquad s_s/g = \frac{41\frac{5}{6}}{120} \doteq \tan 19°13' \tag{2}$$

Thus, crudely:

$$Z = 19°12' \pm 1' \tag{3}$$

A3 But we can improve the precision here by examining[4] ancient rounding even more finely than at §A2: if Pytheas' reading (of his 120-unit-high gnomon) were nearer 41 3/4 or 41 5/6, he would not have rounded to eq.1's 41 4/5. (Ancient unit-division was limited to quarters & sixths for celestial longitudes & latitudes but fifths of degrees were ordinary for meridian-observation based data: e.g., Hipparchos *Comm* [Rawlins 1994L §F4], *Almajest* 7.3.) So the true brackets are the half-way points in the ranges 41 3/4-to-41 4/5 (41 31/40) and 41 4/5-to-41 5/6 (41 49/60), the precise mean of which is (including plus-or-minus found from each difference):

$$\left[\frac{41\frac{31}{40}}{120} + \frac{41\frac{49}{60}}{120}\right]/2 \to \frac{41\frac{191\pm5}{240}}{120} \doteq \arctan 19°12'.2 \pm 0'.5 \tag{4}$$

[2] The Greeks' proclivity for the flawed idea of using an asymmetric gnomon has never been confronted. (Perhaps partly because ancient-astronomy historians tend not to actually try using the equipment they write about.) So, here's a go at resolving the issue: the edge of the penumbral fuzziness of a vertical stake's shadow-tip is not vague. When all but $1'$ of the solar diameter is covered, the remaining sliver of the solar disk's dazzlingly brilliant area is ordmag 1% of the whole, so that such a sliver is ordmag 10000 times brighter than the full Moon — which is why the edge of the penumbra is much sharper and thus more precisely determinable than most expect. Thus, a $1'$ random error is unlikely for careful use of a vertical gnomon. And the experiment is easy to render so precise that the main non-ssd error will be minuscule diffraction. Arrange that the gnomon's shadow be cast into a room protected from non-direct sunlight. Use a vertically-oriented rectangular-plate gnomon (see, e.g., R.Newton 1973-4 p.373 Fig.1). Then, between it and the shadow, bring down another vertically-oriented rectangular-plate until it virtually chops off the solar beam cast upon a flat-horizontal, carefully ruled shadow-measurer. (Due to diffraction, for 5m-high equipment, the gap between shadow-edge & first intensity maximum is c.$1'$; but the uncertainty in that edge's position is smaller: ordmag $1'/10$.)

[3] Subtracting $ssd = 16'$ from eq.5 shows that if Pytheas knew the correct obliquity (but didn't know of the gnomon's ssd-error), he would have thought that his observatory was at about $L = 42°56'$.

[4] We are here taking it for granted that 41 4/5 was Pytheas' original raw datum. (And the original reading would probably have been in shadow/gnomon terms.) Yet we may test the faint possibility that whatever the original reading was, it came to later antiquity as $19°$ 1/5, and only subsequently (in a trig era) was its tangent calculated as a fraction of 120. (But such an assumption itself assumes ancient tangent tables [none have survived] and that these were based upon unit-120, though division of a tabular sine by its complement's sine would cancel their 120-denominators.) However, [a] It seems rather a stretch to suppose that a later ancient would go to such trouble, to turn around the data-reduction process in order to "reconstruct" a lone pseudo-raw datum. Why would such be preserved as special? [b] A firmer objection is that, if Z were $19°$ 1/5, ‡3 eq.15 would not yield its (attested) sum.

Figure 1: Entire Marseilles harbor (Carte Touristique 67 [Marseilles-Carpentras] Institut Géographique National (IGN) France, Paris), including Cape Croisette area (etc) south of the city. Short, narrow east-west white lines mark eq.5's brackets for the latitude of Pytheas' observatory. (Northern bracket's west end is at latter "E" in "CROISETTE"; southern bracket's east end is near southeast tip of Isle de Jarre.) The mainland capes immediately west (off map to left) of Marseilles Bay do not stretch as far south as the upper bracket and so are not potential Pytheas-observatory locations.

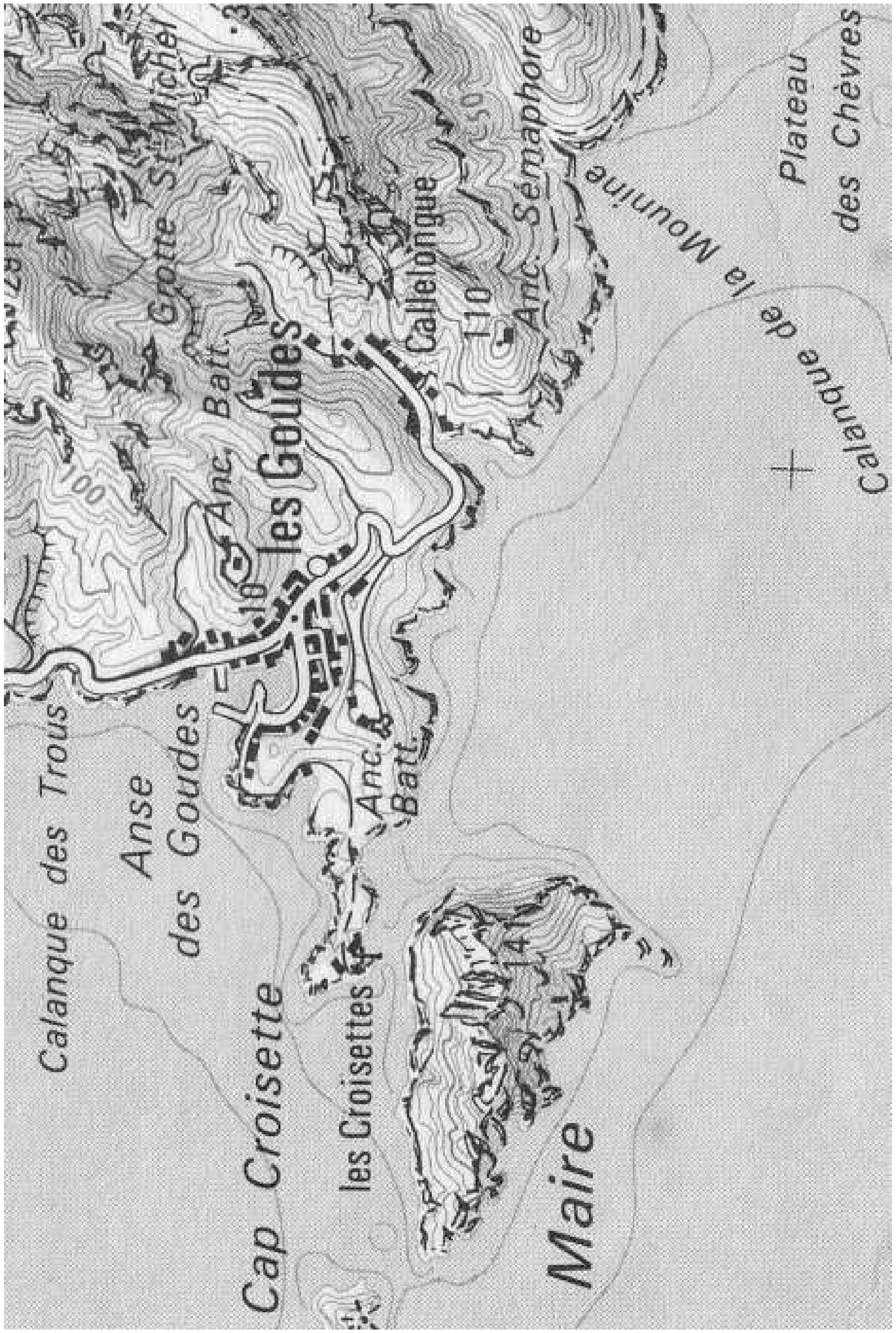

Figure 2: Detail of the region south of Marseilles: Cape Croisette, les Goudes, Callelongue. (Carte de France XXXI 45 [IGN].)

B Finding Pytheas

B1 Now at last we are closing in on the Pytheas observatory's latitude. Using eq.4 and eq.1 we can find the actual latitude L at which Pytheas observed the Sun; the correct empirical relation is (including $ssd = 15'.8$ and r&p = $0'.3$, with [for epoch $-310{\pm}25^{\text{y}}$] obliquity $23°44'.0 \pm 0'.2$, error from uncertainty of Pytheas' exact epoch):

$$L = 19°12'.2(\pm 0'.5) + 23°44'.0(\pm 0.'2) + 15'.8 + 0'.3 = 43°12'.3 \pm 0'.7 \qquad (5)$$

We ignore rms, instead looking for the maximum additive range of errors that are not at all likely to be exceeded if the measurement was indeed carefully *and repeatedly* carried out. I.e., our treatment here is not based upon Gaussian statistics but upon Greek rounding's implied precision, as expressed in eq.4: producing a simple bracket instead of a bell-curve. Eq.5's bracket is obviously from $43°11'.6$ to $43°13'.0$ and is drawn in pale lines upon Fig.1.

B2 We are not the 1st to compute a latitude similar to (if not exactly equalling) eq.5. But previous investigators merely concluded: well, Marseilles is at $43°.3$ N, so Pytheas was only $0°.1$ off the mark — OK-not-bad-and-end-of-story.

B3 But let us instead pay attention to some previously neglected points.
[a] Pytheas' clear precision was $\pm 0'.5$ (eq.4), not §B2's $\pm 0°\,.1$ (c.10 times looser).
[b] The actual possible accuracy for a plain meridian observation has a similar error-bracket. On these bases, DR proposes accepting the theory that the measurement (with the error indicated in eq.5) was as accurate as its precision — and then investigating whether there is independent confirmation that it has provided virtually the ***exact*** latitude of Pytheas' observatory.

B4 Obvious next step: we **check modern maps**[5] of the Marseilles (Massalia) region: Figs.1&2. And we thus find that the best spot an ancient astronomer could have picked near Marseilles is a few miles south of it (Fig.1), the southern part of a peninsula now called Cape Croisette. Its southern coast offers an observatory-dream ***unobstructed southern vista over water***. (Like Tycho's equally well-chosen observatory at Hvin; similarly, Eudoxos' at Knidos and [*DIO 4.1* ‡3 §E] Hipparchos' at Cape Prassonesi [the southern tip of Rhodos] for his southern stars.) Central novel realization here: *the southern part of the Cape Croisette peninsula is a far better location for an astronomical observatory than Marseilles itself, which (Fig.1) faces* **westward** *on the water*. And what is Cape Croisette's location? It is at latitude $43°.2$ N (longitude $5°.3$ E) which neatly matches that found via eq.5 from Massalian Pytheas' S.Solst observation.

C Exploring for As-Yet Impossible Exactitude

C1 We can enjoy further speculation by asking what an astronomer would be looking for in this region. Note (Fig.2) that the easy coastal road, over pretty flat terrain (today called Boulevard Alexandre Delabre), runs into un-negotiably steep coast and mountains about where the Cape Croisette coast turns the corner and starts trending eastward instead of southward. An attractive prospect for the Pytheas observatory's location is on the tiny spit of land that is the extreme west extension of Cape Croisette: a wide hill, about 50m high[6] — almost exactly the height of Tycho's observatory — just high enough to not-infrequently be above the nocturnal aerosol layer.[7] It is marked on Fig.2 as having been the site of "Anc. Batt." (old battlements). Despite its modest height, the hill has a flat water horizon to the south and of all the likely prospects considered here for Pytheas' location, this would have

---

[5] As with *DIO 14* ‡3 §F's discovery (www.dioi.org/gad.htm#blsl) that the Blest Isles were the Cape Verde Islands (not the Canaries, the longtime traditional guess), one wonders why no one previously ever just checked a map and published the obvious solution.

[6] The topo-curves are at 20m intervals for each of the accompanying maps here.

[7] Our thanks again to Nels Laulainen for his 2000-2001 expert advice to *DIO* on such matters.

Figure 3: Panoramic view of Maire Island from very nearby W tip of Cape Croisette.

been the most easily accessible for his Marseilles students or clients. (Cape Croisette would also be an apt location for a sailor-explorer: right on the Mediterranean.)
We next check out a few other candidates.

C2 On a sharper hill to the east (just south of the town of Callelongue), there is an antique semaphore-station marked on Fig.2 (over 100m high) at[8] $43°12'38''.7$ N, $5°21'21''.1$ E, just beyond the end of the extended easy (non-mountain) road from Marseilles to Cape Croisette (i.e., Delabre Boulevard).

C3 As Pytheas was a sailor, we must also consider the possibility that he (like Tycho) operated on an island. The most obvious choice would be tiny but spectacular-gradient Maire Island (whose highest peaks exceed $450'$), which is literally throwing-distance from the west spit of Cape Croisette. (See Fig.2.) Maire's southern coast, though partially quite steep (and not [now] conveniently accessible from Cape Croisette without boat), has the best viewing of any likely[9] location considered here. If Pytheas' 120-unit-high gnomon was 120 Greek *feet* (a Greek foot being $12''1/7$ in modern measure), the high, steep cliffs of Maire (Fig.3) might allow a mostly natural gnomon of such height (which would ensure negligible imprecision from diffraction): the gnomon's verticality verified by plumb-line with a bob dense enough to minimize wind-influence, and the shadow-surface's horizontality verified by use of a water-filled hose. A direct exam of Maire's topography could determine whether this would be feasible.

C4 And there are a few other islands which might be mentioned as possibilities: Tiboulen, de Jarron, de Jarre. All these places' latitudes are easily consistent with the limits of §B1's eq.5. Recall that we began investigating this region due to those very same mathematically-derived latitude limits — and only subsequently noted potential confirmation when finding (§B4) that this put us exactly at the observatory-friendly clear-southern-view coastal region that was nearest Marseilles by road.

C5 Does that striking coincidence assure us that the Cape Croisette region is where Pytheas made his observations? — including the miraculously extant Summer Soltice s_S/g. Hopefully, an archaeological miracle will someday discover the exact spot where stood the scientific home of legendary astronomer-explorer Pytheas of Marseilles.

Acknowledgements: for expert assistance in locating materials, etc, I thank Keith Pickering and Jim Gillispie.

[8] The (over)precision here is c.10 ft. Atop the hill today, Microsoft maps show a lone building which is at least twice 10 ft across.

[9] Maire Island's peak would have even better seeing than its south shore (far lower aerosols on many nights), though with the same extreme isolation-inconvenience that presumably kept Hipparchos from using Mt.Atabyron on Rhodos Island. (Mountain astronomical observatories are a modern phenomenon, due to influence of atmospheric unsteadiness in a telescopic era.)

References

Almajest. Compiled Ptolemy c.160 AD. Eds: Manitius 1912-3; Toomer 1984.
Hipparchos. *Commentary on Aratos & Eudoxos* c.130 BC. Ed: Manitius, Leipzig 1894.
MuJHA 2002. JHA *33.1*:15
Karl Manitius 1912-3, Ed. *Handbuch der Astronomie* [*Almajest*], Leipzig.
R.Newton 1973-4. QJRAS *14*:367, *15*:7, 107.
R.Newton 1977. *Crime of Claudius Ptolemy*, Johns Hopkins U.
D.Rawlins 1982G. Isis *73*:259.
D.Rawlins 1985G. Vistas in Astronomy *28*:255.
D.Rawlins 1994L. DIO *4.1* ‡3.
Gerald Toomer 1984, Ed. *Ptolemy's Almagest*, NYC.