This ***DIO*** is dedicated to the ever-treasured memory of Charlie Kowal (1940-2011) — irreproducible genius, celestial&historical discoverer, brave&principled friend.

‡1 Archimedes' Hidden Measure in Degrees — Sunsize Disguise: His Solar Diameter

Hellenistic Astronomers' High-Empiricism Confirmed by Accurate Solar Brackets

Babylonian Degree-Measure Already Greek-Adopted by 3rd Century BC

A Summary

DIO has recently discovered that Archimedes' *Sandreckoner* estimate of the Sun's diameter — which has the surface look of a crude, pedant-conventional unit-fraction range — was in truth a professional-level empirical measure, expressed instead in the sexagesimal convention of 3rd century BC astronomers: $30'\pm 3'$. Accurate. And couched within reasonably-cautious — and correct — uncertainty-limits. Ultimate resolution below at §E.

B Did 3rd Century BC Greek Scientists Use Degrees? Dating Strabo's Nile Map

B1 In recent years, the superficially ambiguous evidence regarding when the Hellenistic astronomical tradition adopted Babylon's sexagesimal measurement of angles in degrees, arcmin, etc, has led several able, prominent scholars (including sometime-Princetitutees) to doubt or cite doubt that 3rd century BC Greek astronomers used degrees (Dicks 1966 n.15; Jones 1991M n.5; Goldstein & Bowen 1991 pp.103-105; van Brummelen 2009 p.33 n.2).
B2 By contrast, *DIO* has repeatedly pointed out (e.g., Rawlins 1991W fn 53, Rawlins 1994L fn 41, & Rawlins 2008R fn 24) the probability of degrees' use by said astronomers.
B3 But, ironically, DR's own Rawlins 1982N paper perhaps added to the confusion since it showed that the Eratosthenes Nile Map relayed by Strabo 17.1.2 used (instead of degrees) successive halvings[1] of circle-fractions. The exact date of the map is not known; however, the map obviously (Rawlins 2008Q eq.11) post-dates the Alexandria Lighthouse — and pre-dates the end of Eratosthenes' career. Which sandwiches the map into the period c.270-200 BC, dating it for the 1st time. The Nile Map's unit was a Pharos-based (Rawlins 2008Q) Earth-radius probably due to the Alexandria Lighthouse's architect, Sostratos:

$$\text{Sostratos Earth circumference } C_{\text{S}} = 256000 \text{ stades} \tag{1}$$

which is 19% too high — but for Pharos-based Earth-measure, *we expect 20% excess due* to air's bending of horizontal light-rays (to a curvature equal to 1/6 of that of the Earth's surface), so the tight match evidences high precision (half-percent: *ibid* §I3) empirical measurement by Greek scientists (*ibid* §K4), Details at *op cit*.

[1] The Nile Map is based upon successive halvings of $7^\circ 1/2$. Note parallel at ‡3 fn 16.

B4 Eq.1's C_S was ultimately adopted by Eratosthenes but manipulated to (Strabo 2.5.7):

$$\text{Eratosthenes Earth circumference } C_\mathrm{E} = 252000 \text{ stades} \tag{2}$$

— the canonical "Eratosthenes" circumference, his slight alteration (less than 2%) perhaps effected to arrange an even multiple of 360, so that for Eratosthenes

$$1^\circ = 700 \text{ stades} \tag{3}$$

suggestive[2] of Greek scientists' adoption of the degree at least by the mid-3rd century BC.

C Earlier Evidence

C1 When we began looking for evidence bearing on the era when Greeks started precisely measuring celestial coordinates in degrees, the earliest datable clue encountered was Ptolemy's collection of 18 stellar declinations — 12 by Timocharis & 6 by Aristyllos — observed in Alexandria c.300 BC & c.260 BC, resp (Rawlins 1994L Table 3) and all expressed in degrees at *Almajest* 7.3. Particularly striking is the uniform rounding by Aristyllos of all six of his star declinations to $1^\circ/4$ precision: a conscientiously cautious practice — perhaps intended to avoid erroneously reporting slightly discrepant empirical results, but which may (Standish 1997 DR Comm. §G12) have cost him discovery of precession, an honor which (Rawlins 1999 §D5) should instead go to his contemporary, Aristarchos. The obvious question regarding Aristyllos: if his declinations were originally reported in some other measure than degrees, how likely is it that, after hypothetical subsequent transformation (into the degree-values reported by Ptolemy at *Almajest* 7.3), all six data would[3] end up exhibiting consistency with quarter-degree rounding?

[2] Early Greeks divided Earth-circumference C into 60 parts (Strabo 2.5.7), not the public-treatise custom of quadrant-unit-fractions (Aristarchos&Archimedes: *idem* or our eq.6). [Note added 2014/4/29:] So ***was the stade defined sexagesimally***? Like our nautical mile (C/21600)? Or the meter (C/[4x10^7])? Greeks expressed fractions sexagesimally. Was the formerly-unsteady, locally-varying stade imperially regularized c.300 BC as 1/60th of 1/60th of 1/60th of 1^C? Above-***attested*** $1^\mathrm{C}/60$: cascade's *determinant step 1*. Our proposed ***integral unit-fraction*** in modern sexagesimal notation: 1 stade ≡ PRECISELY 0^C;00,00,01 (thus $1^\circ \equiv 600$ stades). At Egypt, latitudinal Earth-curvature's $C \doteq 39{,}900$ km: $C/60^3 = 184.7$ m $\doteq$ standard Alexandrian 185 m stade (185 m independently-validated: Rawlins 2008Q §K2). Easy *thus frequent* testing confronted scientists with Pharos-vs-sunsets Earth-measure-discord's reality (*idem*): did they suspect (quantify?) atm-refraction & geometric-mean the 2 methods' results to find 185 m? Or much less speculatively (given Pharos-erection's far greater labor): did a few-hundred-miles version of the unsubtle but low-systematic-error Kleo-Method (Rawlins 2008Q §A4; *Geogr.Dir.* 1.3.2-3), or a royal surveyors' project, find equivalent of correct $C = 39900000$ m, thus 1 stade $\equiv C/60^3 = 185$ m, well before Sostratos&co got clever with the Pharos? See www.dioi.org/cot.htm#csqm.

[3] We can test independently whether Aristyllos (c.260 BC) used 60ths (fn 2) instead of 360ths of a circle for his share of the only 18 Greek declinations surviving from the 3rd century BC: those 6 in the north quarter of the sky. Try his ζ UMa declination, where *Almajest* 7.3 reports $\delta = 67^\circ 1/4$, which — in circle-60ths — exactly translates to $11^\mathrm{x}5/24$ (not credible) or $11^\mathrm{x}12'1/2$ (too precise, compared to the simplicity of $67^\circ 1/4$). Testing instead $11^\mathrm{x}12'$ & $11^\mathrm{x}13'$, we find that they would result in $67^\circ 1/5$ (or 1/6) & $67^\circ 1/3$, respectively — not $67^\circ 1/4$. So this star alone eliminates the theory that Aristyllos used degree-60ths. Goldstein & Bowen 1991 pp.103-105 suggest he could have used two-degree "cubits" or degrees — or half-degree "points" or half-degree "Moon-breadths". But the differences between these measures are too trivial to regard as generic. Also: [a] two-degree cubits (*ibid* p.104) would require overprecise 13 1/2 & 16 1/2 for Archimedes' solar brackets; and even more unlikely: 33 5/8 for Aristyllos' ζ UMa declination. [b] Ancient mention of points is later than of degrees. [c] As for Moon-breadths ($1^\circ/2$): these were just visual yard-sticks for eyeball-observers who lacked the ringed astrolabe — irrelevant to Aristyllos' transit-instrument observations. Further: Moon-breadths are also Sun-breadths. Wouldn't it seem a mite superfluous for Archimedes to announce that he had empirically measured the Sun's width to be one Sun-width? (And: how do you call a quantity equal to itself plus-or-minus 10%?)

C2 The next point coming to our attention (noted at Rawlins 2008R fn 24) was Aristarchos' record: his 1/720 of a circle for solar-diameter (eq.7; Rawlins 2008R eq.3), which is $1^\circ/2$; his empirical estimate (his Hypothesis 4) that half-Moon elongation was 1/30 of a RtAng from quadrature (*ibid* eq.4 or Heath 1913 pp.352-353), which is 3°; and Ptolemy's mention (*Almajest* 4.2) that in the Aristarchos luni-solar scheme (Rawlins 2002A), the saros' excess over 18^y was $10^\circ 2/3$ — that is, 32° excess for the 54^y exeligmos — an integrality which led to *DIO*'s reconstruction (*idem*) of the origin of Aristarchos' monthlength:

$$M_\mathrm{A} = 29^\mathrm{d}31'50''08'''20'''' = 765433^\mathrm{d}/25920 = 765433^\mathrm{h}/1080 \tag{4}$$

(falsely labelled by hist.astron's political-centrists the "Babylonian" month though unattested in Babylon before c.200 BC), based upon the 4267-month eclipse-cycle (as correctly reported by Ptolemy at *Almajest* 4.2), a value accurate to a fraction of a timesec then and now. (For other viewpoints, see, e.g., Swerdlow 1980 & Engelson 2006A.) That's one part in several million, and it's based upon degrees not only in the expressions for saros & exeligmos but — as pointed out to DR by John Britton and John Steele — in the key rounding (Rawlins 2008Q §A8) that produces the precise degree-expression of Aristarchos' monthlength in its original form:

$$M_\mathrm{A} = 29^\mathrm{d}191^\circ 00'50'' \tag{5}$$

which later became equivalently expressed in the sexagesimal format (eq.4) we know from *Almajest* 4.2.

D Archimedes the Astronomer

D1 Archimedes is not usually seen as astronomer but as combo of mathematician and arms-designer. Yet the latter career could hardly have occurred without a scientist's knowledge, drive, and thought-habits.

D2 *Almajest* 3.1 quotes Hipparchos' testimony that he & Archimedes observed solstices to an accuracy no worse than $1^\mathrm{d}/4$, so we know that Archimedes had 1sthand outdoor experience in solar work.

D3 His measure of the Sun's diameter (to be analysed in what follows) gives flesh to that supposition, as well as providing a prime example of ancient scientific writers using circle-fractions for publication of empirical data actually measured in degrees, a more familiar example of which is Eratosthenes' description of the Earth's obliquity as 11/83 of a semi-circle, when (*Almajest* 1.12) $23^\circ 51'1/4 \pm 1'1/4$ was the actual (precise but inaccurate: Rawlins 1982G eq.9) mean measurement by asymmetric (unfortunately) gnomon.

D4 Archimedes' report (Archimedes p.224) is that the Sun's angular diameter $d_\odot$ is between 1/200th and 1/164th of a quadrant — a right angle or 90°. Which can be expressed thusly:

$$\text{Archimedes:} \quad \mathrm{RtAng}/200 < d_\odot < \mathrm{RtAng}/164 \tag{6}$$

What can a description of such oddity (§E1) be telling[4] us?

[4] However, one must be taught BY evidence rather than teaching TO it. E.g., Shapiro 1975 p.77 long ago realized that exact conversion of Archimedes' brackets equalled $27'$ & $32'56''$. But he then spurned the discovery-opportunity here by neglecting to ponder: [a] the latter angle's glaring nearness to $33'$, [b] the pair-average's nearness to Aristarchos' $1^\circ/2$ (reported by Archimedes in the same opus under examination), or [c] the wisdom of computing to 4 places using a 3 significant-digit number! (The same naïvete affected even Delambre 1817 [1:104], but that was back in an era when scientists were insensitive to significant-digits.) So Shapiro didn't calculate in reverse by simply (§E1) hypothesizing & checking to see whether $5400'/33'$ rounded to 164 or to a different integer. He instead swiftly concluded by just echoing establishment dogma:

"the degree was, of course, not a unit used by Archimedes".

(If anyone among our readers knows of an earlier analyst who realized the sexagesimal truth behind Archimedes' solar brackets, please inform us so that we may add a citation here in this issue's next printing.)

E Archimedes' RtAngle-Unit-Fractions: His Solar Diameter Solved

E1 Since Archimedes' predecessors, Aristarchos (c.280 BC) & Aristyllos (c.260 BC), & likely Timocharis (c.290 BC) measured astronomical angles in degrees (see fn 3 & Rawlins 2008R fn 24), let us investigate eq.6 by the hypothesis that it expresses an empirical range, originally in degrees. Archimedes is our sole reliable witness to Aristarchos' solar diameter $d_\odot$, making it 1/720[th] of a circle or a half-degree (§C2; Archimedes p.223):

$$\text{Aristarchos:} \quad d_\odot = 30' \tag{7}$$

In eq.6, the number 164 is peculiar (prime factors: 2 & 2 & 41, which lead nowhere), so we are inspired to dig beneath the surface. Noting additionally that 200 and 164 differ by c.20%, we try the following hypothesis for explaining eq.6:

$$\text{Archimedes:} \quad d_\odot = 30' \pm 10\% = 30' \pm 3' \quad \text{or} \quad 27' < d_\odot < 33' \tag{8}$$

To test for confirmation we convert eq.8 into not only circle-fraction format (§D3) but specifically into satisfaction of another ancient schoolbook convention, *unit-fractions* (inverse-integers — as displayed in eq.6); we find the unit-fractions' denominators by dividing $27'$ & $33'$ successively into RtAng $= 5400'$, yielding 200 & $163\frac{7}{11}$, resp, which (after integralization of the latter) produces:

$$\text{Archimedes:} \quad \text{RtAng}/200 < d_\odot < \text{RtAng}/164 \tag{9}$$

— the very Archimedes bracket-expression (eq.6) we'd set out to trace the origin of. Being the least ambiguous of all entries in the list of evidences for 3[rd] century BC use of degrees, our finding now takes its place at the head of the list. And, with respect to Archimedes-as-astronomer (§D), eq.8 is absolutely accurate: the Sun never strayed outside limits $31'$-$33'$, so not only was his solar diameter correct but his brackets were judiciously applied.

E2 From fn 2, eq.8, & Rawlins 2008R fn 24, we see that, starting from dividing Earth C into 60 parts c.300 BC, Greek science transitioned to degrees early in the 3[rd] century BC.

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