‡3 Hipparchos' Fake −381/12/12 Mis-Eclipse

His Eclipse Calculations Used Pairs — Not Threesomes
Newly Confirmed By Resolving His One-Degree Fudge
Hipparchan Computations' Mechanical Flawlessness
Greek Invention of Order-of-Magnitude Estimation

A Summary

A1 Hipparchos' work with eclipse-trios (in the 140s BC) was mathematically analysed in 1991 by *DIO*, and all[1] 4 of the lunar-orbit elements (*Almajest* 4.11) Hipparchos had published were precisely elicited thereby at Rawlins 1991W, www.dioi.org/vols/w13.pdf, eqs.19-20&23-24, as generously noted in the History of Science Society's *Isis* during its coverage (Thurston 2002S) of *DIO*'s reconstructions. These 4 solutions transpired through far simpler analysis *and via more ancient-style round-number elements* than prominent prior work (§A2) that failed to reproduce the same 4 data.[2] Analysis' by-product: revelation that Greek science used order-of-magnitude. [DR thanks John Britton & the late Hugh Thurston for thoroughly and expertly verifying all of the mathematical steps of Rawlins 1991W. Also special thanks to Dennis Duke for inspiring, vetting, and tolerating the present paper.]

A2 The 1991 matches are so unanswerably perfect that they have never even been cited by the history-of-astronomy political center, the esteemed "Muffia", which clings to its own goofy old theory (Toomer 1973), though it fits none of the 4 above-cited elements. (Unless one blatantly funnies input data: §K.) Nor does this cult-fave (§§C1, H2, I9&I10) theory explain Hipparchos' 1° data-fudge, a 2000^{y}old puzzle 1st solved here at §G1 by extension of the gratifyingly fruitful 1991 analysis, which also bears a glimmer of early heliocentrism.

A3 Below, we precisely solve (§§C2-G) both trios, achievable because ***Hipparchos' calculations are always mechanically flawless*** (a point helping place his observatory near Lindos: see *DIO 7.1* ‡3 end-Note), our historically key hitherto-implicit finding (Rawlins 1991W, confirmed: Rawlins 2009S Table 2) — ever-denied (e.g., fn 22) by *DIO*-shunners, who can only promote their predictable (Rawlins 1991W §H2 [g]) desperately weird anti-*DIO* pseudo-discoveries by dreaming-up[3] Hipparchos (fn 10) & Strabo (Rawlins 2009S §B6) math errors at will. (Details of 30^y-shun's tantrum-origin: see Rawlins 1991W §B.)

B Hipparchos' Data

B1 Our subject here will be two much-discussed ancient lunar eclipse trios: from 383-382 BC (observed in Babylon) and 201-200 BC (observed in Alexandria). The trios are today generally designated as "Trio A" & "Trio B", respectively. (All six dates listed at fn 5.) Both trios were mathematically analysed by Hipparchos c.150-145 BC, during his primitive attempts to improve knowledge of the Moon's nonuniform motion. The empirical data he started with were merely past reports of eclipses' times (& magnitudes & durations), for which he computed true longitudes of the Sun (thus Moon opposite) from his solar tables of the moment.[4] Hipparchos' stated intervals are for two pair from each trio,[5] as follows:

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[1] The Hipparchan numbers to be (re)traced here are: 3144, 3122 1/2, 327 2/3, 247 1/2. Rawlins 1991W solved all four to precision given. The paper's calculated reconstructions are reprised below: 3144 (eq.5), 3122 1/2 (eq.6), 327′39″ (arrowed A3-A2 at §G2), 247′30″ (arrowed B2-B1 at §F2).

[2] Rawlins 1991W §P2: all 4 unaltered Muff-nonfits compared sidebyside with *DIO*'s 4 neat matches.

[3] Sociological background to such's inevitability (*DIO 4.3* ‡15 §G9): banishers are unwittingly *gambling* — risking their reputations irrevocably on the improvident demand that the pariah is *permanently valueless*. Since no blackballing archon can admit to jailing valid ideas, the exiled journal can't ever be credited for making a single discovery. So each time it does, its bet-redoubling shunners **must keep publicly faking its accumulating achievements' worthlessness** (welcome exception: ‡2's fn 10 on its eq.4). See, e.g., *DIO 4.2* ‡9 §T, *DIO 6* ‡3 §B2.

[4] Hipparchos' adopted solar orbit varied from time to time, as we saw at ‡2 §O.

[5] In temporal order, we call Trio A's eclipses: A1 (−382/12/22-23), A2 (−381/6/18-19), A3 (−381/12/12-13). Trio B analogously: B1 (−200/9/22-23), B2 (−199/3/19-20), B3 (−199/9/11-12).

B2 The Trio A time-intervals and longitude-intervals (*Almajest* 4.11):

$$A2 - A1 : 177^{\rm d}13^{\rm h}3/4 \;\; 173^\circ - 1^\circ/8 \qquad A3 - A2 : 177^{\rm d}01^{\rm h}2/3 \;\; 175^\circ + 1^\circ/8 \quad (1)$$

The Trio B time-intervals and longitude-intervals (*idem*):

$$B2 - B1 : 178^{\rm d}06^{\rm h} \;\; 180^\circ 20' \qquad B3 - B2 : 176^{\rm d}01^{\rm h}1/3 \;\; 168^\circ 33' \quad (2)$$

C Precisely Solving the Origin of Hipparchos' Lunar Distances

C1 *Almajest* 4.11 supplies Hipparchos' disparate results for lunar distance R and eccentricity e (or, equivalently,[6] epicycle-radius r):

$$\text{Trio A}: \qquad R = 3144 \qquad e = 327\ 2/3 \quad (3)$$

$$\text{Trio B}: \qquad R = 3122\ 1/2 \qquad r = 247\ 1/2 \quad (4)$$

For decades after Toomer 1973, two of these desiderata, the lunar-distances R (3144 & 3122 1/2) were held by the political center (traditionally overlapping the O.Neugebauer klan) to be by-products of elaborate calculations, *during* which the prime sought-numbers *occur* and are then frozen-in-midstream: i.e., *the very yardstick-radius of the lunar orbit* (R) is supposed to just fall out of the process on-the-fly (§§I8-I9 & §§I12-I13) — unheard-of in Greek (or any other) mathematical astronomy. Including Indian.

C2 Rawlins 1991W precisely reproduced the R values by showing both were instead *fixed at the outset*, finding 3144 by just applying simple trig to Aristarchan data (Hipparchos was partial to such: *ibid* fn 243 & §O8): half-Moon elongation 87° (or 3° from quadrature: ‡1 §C2), and Sun at distance 1000 Earth-radii (§D). From *ibid* eq.23, we have:

$$R(\text{Trio A}) = 1000^{\rm e}\cot 87^\circ = 1000^{\rm e}\tan 3^\circ \doteq 52^{\rm e}24'28'' \doteq 3144' \quad (5)$$

Perfect match to the attested value for Trio A's R (fn 1). Notice that this R has thus established a startling revelation: measuring the lunar distance in solar-based units is the mark of a heliocentrist. And recall that all input data in eq.5 came from Aristarchos, who was the public pioneer of heliocentrism. (See also fn 6.)

C3 True, eq.5 doesn't yield Trio B's $R = 3122\ 1/2$. But instead of an impediment, this discord is about to be revealed as the clincher for eq.5's source.

C4 One of the commonest ancient and modern misreadings of Greek numbers is the confusion[7] of sixtieths with fractions. Evidently Trio B's computer misread Trio A's $52^{\rm e}24'$ (eq.5) as $52^{\rm e}1/24$, thus (as 1st discerned at Rawlins 1991W eq.24)

$$R(\text{Trio B}) = 52^{\rm e}1/24 = 52^{\rm e}02'1/2 = 3122'1/2 \quad (6)$$

— the precise attested value for Trio B's R (eq.4; fn 1). This delightful confirmation of eq.5's heliocentrist revelation boosts our certitude that heliocentrism — lethally suppressed though it was (Rawlins 1991P) — carried on quietly in the astronomical community (as it did in 18th century France), even showing up in the work of geocentrist Hipparchos.

No absolute Hipparchan value of any's hour or longitude survive explicitly. (Strictly differences: *Almajest* 4.11.) But all of these dozen absolute data were precisely reconstructed at Rawlins 1991W §§M9-10 & L2-3.

[6] Is it indicative that Hipparchos started with the eccentric lunar theory, rather parallel to the heliocentrists' model for planets, but later moved over to the epicyclic lunar theory, parallel to the geocentrists' model? Note it was the earlier (Trio A) computer who introduced (§C2) heliocentrist measure into determining R.

[7] For examples, see Rawlins 1991W §O3. Also Rawlins 1994L ‡3 fn 39.

D Ordmag's Debut: Distance to the Sun

In the 2012 June *Astronomy* (p.31), Bill Andrews asks: when historically did the idea of using order-of-magnitude (ordmag) arise? The answer resides in our eq.5: Greeks pioneered adoption of order-of-magnitude, naturally resorting to powers of ten to gauge the too-uncertain-for-precision solar distance in Earth-radii. Remote from their subjects, astronomers were the inevitable inventors of ordmag. Sun-distance estimates by eminent ancient scientists follow (superscript e = Earth-radii). Eratosthenes: $100^{\rm e}$ (Rawlins 2008Q eq.12); early Aristarchos & mid-career Hipparchos: $1000^{\rm e}$ (Rawlins 2008R §D1; above eq.5); late Aristarchos, Archimedes, & Poseidonios: $10000^{\rm e}$ (Rawlins 2008R eqs.14-15).

E Exact Origins of Hipparchos' Eccentricity e & Epicycle-Radius r

E1 For each eclipse-pair, *Almajest* 4.11 provides two pair of data (already listed here at §B2): time-interval Δt in days and true longitude-interval $\Delta\phi$ in degrees; Δt is multiplied times mean lunar motion — namely, Kallippic solar motion (Rawlins 1991W §§K9&M4) plus long-canonical Aristarchan (Rawlins 2002A) synodic lunar motion (‡1 eqs.4&5) — which yields Δf, the mean lunar motion in degrees during the interval between the eclipse-pair. For any pair, non-uniform motion causes a gap between mean-longitude-difference Δf and the true-longitude difference $\Delta\phi$ — where, again: each $\Delta\phi$ is already supplied explicitly (§B2) at *Almajest* 4.11. (Though, Hipparchos had illegitimately [§G4] altered A3's longitude by -1°, as will be shown below: §G1.) This gap is labelled δ:

$$\delta = \Delta\phi - \Delta f \quad (7)$$

True longitude ϕ was computed in each case by Hipparchos from the solar orbit he had adopted at the time of the calculation. (These orbits' poorness created fateful errors in the ϕ values, which were to undo [§G1] the very Pair Method [§E7] Hipparchos used.)

E2 The longitudes ϕ in Hipparchos' famous catalog of $600^{\rm y}$ of eclipses were presumably computed well before his trio-calculations. Said catalog was naturally compiled going backward in time, a judgement which becomes more than guesswork when we find that the -200-199 true longitudes ϕ were calculated from an inferior early "EH" solar orbit[8] whose 4 elements are given at Rawlins 1991W §K9 (along with its empirical bases: *ibid* §§K4-K9), while the parallel -382-381 longitudes were found from a solar orbit that constituted a relic of the later transition from EH to his famous "PH" orbit (preserved at *Almajest* 3.1-7), which is why it bore elements from both EH & PH: a stitched-together element-mix quasi-facetiously called "Frankensteinorbit". (See Rawlins 1991W § M4-M5.)

E3 Determining chronological order (*ibid* §M5): the 2 elements of Frankensteinorbit drawn from PH (solar apogee-at-epoch & mean-longitude-at-epoch) were constants — thus available for immediate use — while the 2 elements drawn from EH (eccentricity & mean

[8] Duke 2008W, *JHA*'s August Pb paper, rejects (‡2 fn 10) all of DR's 3 Hipparchos orbits (EH, Frankenstein, & [Rawlins 1991H] UH), deeming them "neither conclusive nor satisfying" since (emph added) "parameters deduced from trio analyses are very sensitive to small *changes* in the input data" (shouldn't that read "small *errors*"? — see ‡2 fn 10 items [4]-[5]), from Duke 2008W's unique delusion (‡1 §B4) that Greek solar data averaged $15'$ error. Only citation relating to target DR is nonexplicit: a *JHA*-doctored note; see *DIO 6* §§D1&H2 & fn 20. (*JHA* refereeing. Again.) But uncited Rawlins 1991W fn 205 explored this sensitivity, *thus DR didn't just compute orbit-elements from trio* ϕ but the reverse: EH&UH were instead initially founded upon Hipparchan cardinal-point data (firm or reasonably reconstructed: ‡2 §§K or Rawlins 1991W §K), then *tested against* extant trios' ϕ. Further testing found that a meld (*correctly ordered, chronologically*: §E3) of EH&PH fit Trio A's ϕ, thereby establishing Frankensteinorbit & dating it to the -145 [V.Equinox] (‡2 §O3). Doubting UH requires rejecting ‡2 eqs.3-4. (Contra pp.23-24 of the very Jones 2005 paper cited by Duke 2008W p.289 n.9.) UH — incl. above ‡2 eq.4 — solved *five* mysteries simultaneously (Rawlins 1991H): [a] why Aristarchos & Hipparchos solstices are (‡2 §C1) sole hourless *Almajest* 3.1 Sun data; [b] all 3 Trio C ϕ (*Almajest* 5.3&5); [c] $5'$-PH-discrepant f of Trio C's 2nd ϕ; [d] 0°.2 amplitude of AncStarCat zodiac stars' periodic error; [e] Moon-phase when AncStarCat fundamental stars observed. (Also, suggestive: Hipparchos' UH&AncStarCat -127 A.Eqx epoch follows Meton's S.Solst by $304^{\rm y}1/4$, ***exactly*** 1/16th [Rawlins 2002A fn 17] of Hipparchos' $4868^{\rm y}$ Great Year: ‡2 §P4.)

motion) required tables for usage, ***which would take time to compile***, thus Hipparchos' continuing use of the old EH tables for a little while after establishing the PH orbit. This situation is consistent (§E2) with the −382-381 longitudes (for the early eclipse catalog, long before the analyses of trios) being computed later than the −200-199 ones — presumably c.−145, the time when (‡2 §O3) he established the iconic (‡2 §O1) PH orbit.

E4 Continuing in Hipparchos' footsteps, we now compute (eq.7) the δ for all six eclipse-pairs. (Notice that for each trio, the sum of the three δ is zero.)

$$\begin{aligned}
\delta_{\text{A2-A1}} &= 172°53' - 179°46'07'' = -6°53'07'' \\
\delta_{\text{A3-A2}} &= 176°07' - 173°08'04'' = +2°58'56'' \\
\delta_{\text{A1-A3}} &= 11°00' - 7°05'49'' = +3°54'11'' \\
\delta_{\text{B2-B1}} &= 180°20' - 188°41'24'' = -8°21'24'' \\
\delta_{\text{B3-B2}} &= 168°33' - 159°46'31'' = +8°46'29'' \\
\delta_{\text{B1-B3}} &= 11°07' - 11°32'05'' = -0°25'05''
\end{aligned}$$

E5 Via likely roundings, Rawlins 1991W (§§M9&L2) reconstructs Hipparchos' absolute times of the six eclipses. Hipparchos' mean anomaly for each eclipse was found through multiplying his traditional Aristarchan lunar mean anomalistic motion (*ibid* eqs.6-7: not quite the same as *Almajest* 4.7's) by the time since his theories' epoch (Phil 1 = −323/11/12: ‡2 §O3), and then adding the result to his equally Aristarchan lunar mean-anomaly-at-epoch $g_\circ = 82°$ (Rawlins 1991W eq.9),[9] which relates to apogee-at-epoch $A_\circ$ thusly:

$$g_\circ = \epsilon_\circ - A_\circ = 178° - 96° = 82° \tag{8}$$

E6 This produces the following mean anomalies:

$$\begin{aligned}
g_{\text{A1}} &= 224°1/3 & g_{\text{B1}} &= 297° \\
g_{\text{A2}} &= 24°1/3 & g_{\text{B2}} &= 105°5/6 \\
g_{\text{A3}} &= 177°44' & g_{\text{B3}} &= 246°
\end{aligned}$$

E7 Ere 1991, all presumed (from *Alm* 4.5) Hipparchos used eclipse-trios to find 3 unknowns simultaneously: e, $g_\circ$, & $\epsilon_\circ$. Rawlins 1991W §§ N5f found he used not trios (which produce no matches: §A2) but eclipse-*pairs* (which do: §§F2&G2), thereby seeking only e, while appropriating $g_\circ$ (see Rawlins 1991W §N10!!) and ultimately it seems (§F4) $\epsilon_\circ$ from a prior astronomer: eq.8. [A paper (§K1) rejecting the Pair Method cites *Alm* 4.5's belief that Hipparchos' method is that of *Alm* 4.6&11, meanwhile accepting the paper's own 3438-base alteration of same.] To find e from an eclipse-pair, trigmaster Hipparchos used the pure-trig Pair Method (easier&clearer than Ptolemy's eclipse-trio Simultaneous Method [§I1], though inferior in result):[10] for any eclipse-pair we specify their 2 mean anomalies g (already computed at §E6) as α & β and use them with δ (§E4) in the following 3-step trig procedure (perhaps unknown during the 21 centuries up to Rawlins 1991W §N13):[11]

$$U = -[(\cos\alpha + \cos\beta) + \cot\delta(\sin\alpha - \sin\beta)]/2 \tag{9}$$

$$V = \cos(\alpha - \beta) + \cot\delta\sin(\alpha - \beta) \tag{10}$$

$$e \text{ or } r = R/(U + \sqrt{U^2 - V}) \tag{11}$$

[9] Where $\epsilon_\circ = 178°$ is Aristarchos' (later Hipparchos'&Ptolemy's) lunar mean-longitude-at-epoch; $g_\circ = 82°$, his mean-anomaly-at-epoch; $A_\circ = 96°$, his apogee-at-epoch (epoch = Phil 1: §E5).

[10] The poorness of Hipparchos' results alone suggests a primitivity incongruent with the sophisticated Simultaneous Method. (And the inconsistent consistencies of the 60$^{\text{p}}$-based values of §F2 vs §G2 suggest worse.) As earlier realized by van der Waerden and shown at Rawlins 1991W (§S1) & Rawlins 2009E, Hipparchos wasn't an outstandingly able math-theoretician, though (contra Duke at §§K1&K4 & Jones at Rawlins 2009S §§G2-G3) *an unerringly reliable computer*: here & Rawlins 2009S Fig.1.

[11] This presumes that Hipparchos didn't solve pairs by trial. Note: all §E6's g are round fractions (suggesting that some eclipse-data might've been slightly adjusted), except for the near-perigee (thus *very* sensitive) A3 case, where 1-unknown math (via §F4's equation) upon pre-doctored A3 yields e both outsized & *negative*. (An alternate explanation for Hipparchos' fudging eclipse A3 by −1°.)

F Expanding Pair-Analysis

But we go beyond Rawlins 1991W §N14 by computing e (or r) for all 6 pair, thus adding 4 new data (not in *ibid*) that ultimately reveal (§§G1-G3) the cause of Hipparchos' fake.

F1 The three Trio A results for e, recalling R of eq.5 (& including 60$^{\text{p}}$-based values):

$$\begin{aligned}
e_{\text{A2-A1}}/R_{\text{A}} &= 334'18''/3144' = 6^{\text{p}}23' \\
e_{\text{A3-A2}}/R_{\text{A}} &= 529'06''/3144' = 10^{\text{p}}06' \\
e_{\text{A1-A3}}/R_{\text{A}} &= 272'04''/3144' = 5^{\text{p}}12'
\end{aligned}$$

F2 The three Trio B results for r, recalling R of eq.6:

$$\begin{aligned}
r_{\text{B2-B1}}/R_{\text{B}} &= 247'30''/3122'30'' = 4^{\text{p}}45' \quad \Longleftarrow \\
r_{\text{B3-B2}}/R_{\text{B}} &= 248'35''/3122'30'' = 4^{\text{p}}47' \\
r_{\text{B1-B3}}/R_{\text{B}} &= 261'54''/3122'30'' = 5^{\text{p}}02'
\end{aligned}$$

F3 For Trio B, it looked to Hipparchos like there was sufficient consistency to justify taking any of the values as a good approximation to r. (The consistency was at best luck,[12] given the poorness of the EH orbit producing input ϕ data.) He chose the top line (arrowed B2-B1) to publish as his r solution. It agrees perfectly with his attested Trio B r (eq.4).

F4 [Added 2012 September. Dennis Duke notes: for pre-fixed $g_\circ$&$\epsilon_\circ$, e (or r) is findable from 1 eclipse. Equation: $-1/e = \cos g + \sin g/\tan(\phi - f)$; yet results don't recover attested e&r (§C1), while §§F2&G2 fit. But why did Hipparchos use the harder Pair Method? [a] He (& Rawlins 1991W §N12) knew it finds e (or r), pre-fixing only $g_\circ$ but not yet $\epsilon_\circ = 178°$ [fn 9; eq.8]. (***NB***: Ptolemy's precise-to-arcmin Ant 1 [137.547] solar&lunar $\epsilon_\circ$ descend from Phil 1 [−322.148] round $\epsilon_\circ$ [Rawlins 1991W eq.8].) [b] Results for pairs tend to be more consistent than for single eclipses. [c] A perigee eclipse (A3) isn't useless when paired. [d] Weather blocked most trios' full capture (thus Hipparchos' resort to 200BC & 382BC data?), so pair-analysis pre-fixing $g_\circ$ (or $\epsilon_\circ$) may have been common.]

G Hipparchos Fakes the Impossible for −381/12/12: A 179° True-Elongation Lunar Mid-Eclipse!

G1 But Hipparchos saw that Trio A's results (§F1) for lunar e were dreadfully inconsistent. (Mostly due to Babylonian observational time-errors [Rawlins 1991W fn 223] & to Frankensteinorbit's solar e being more than 50% higher than reality.) So he[13] committed a crime against science: finding that altering ϕ_{A3} by −1° dramatically "saved" the situation (fn 11), he made the alteration, a sleight that changed the Trio A input data (from §E4) to:

$$\begin{aligned}
\delta_{\text{A2-A1}} &= 172°53' - 179°46'07'' = -6°53'07'' \\
\delta_{\text{A3-A2}} &= 175°07' - 173°08'04'' = +1°58'56'' \\
\delta_{\text{A1-A3}} &= 12°00' - 7°05'49'' = +4°54'11''
\end{aligned}$$

G2 Which converted the results of §F1 instead to:

$$\begin{aligned}
e_{\text{A2-A1}}/R_{\text{A}} &= 334'18''/3144' = 6^{\text{p}}23' \\
e_{\text{A3-A2}}/R_{\text{A}} &= 327'39''/3144' = 6^{\text{p}}15' \quad \Longleftarrow \\
e_{\text{A1-A3}}/R_{\text{A}} &= 336'33''/3144' = 6^{\text{p}}25'
\end{aligned}$$

[12] The seeming good luck of Trio B's consistency was bad luck, since it deluded Hipparchos into expecting similar consistency for Trio A; so when it didn't happen, he made it happen: §§F3-G1.

[13] Eq.6's miscue indicates that at least the R of Trio A & Trio B were computed by distinct members of a hitherto (fn 6) hypothetical Hipparchan stable. Note obvious parallel to the problems producing the few faked stars (Rawlins 1992T & Rawlins 1993D) of Tycho Brahe. (Known to have had a stable.)

Unfortunately, Hipparchos chose the 2nd solution (arrowed A3-A2) for publication, a solution *squarely based upon fabrication*. Its e (which of course would be rounded to $327'2/3$) agrees with the attested Hipparchan value: eq.3. [Note added 2012/9/16. For both e&r, he picked the value *nearest a round fraction* ($327\ 2/3 = 6^{\mathrm{p}}1/4$, $247\ 1/2 = 4^{\mathrm{p}}3/4$): which backs *DIO*'s theory (e.g., eq.8, fn 11, Rawlins 2002A §A6) that ancient astronomers preferred round numbers for elements. (And for observational data: R.Newton 1977 pp.250-254. Newton's discovery clinched by $30'$ endings: *DIO* 2.3 ‡8 fn 47 & Rawlins 1994L fn 5.)]

G3 Notice that we now not only have the why of fabrication but additionally have shown that it arises out of the theory that Hipparchos was mathematically investigating the lunar orbit via pairs not trios. The foregoing is therefore a surprise vindication — unanticipated in Rawlins 1991W — of *idem*'s Pair-Method explanation of the curiously skimpy (& explicitly-in-pairs) early data which Hipparchos&Ptolemy left us via *Almajest* 4.11.

G4 Check the 179° difference between Hipparchos' pre-existing (eclipse-catalog: §E2) Frankenstein-orbit-computed *solar* $\phi_{\mathrm{A3}} = 257°7/8$ (Rawlins 1991W §M10) and the subsequently $-1°$-fudged *lunar* $\phi_{\mathrm{A3}} = 76°7/8$ (*ibid* §N15) — a forgery which naturally also shifted contingent $\delta_{\mathrm{A3\text{-}A2}}$&$\delta_{\mathrm{A1\text{-}A3}}$ from their §E4 values to the 1°-altered values of §G1, which underlie §G2. [Hipparchos didn't recompute (§B1) his eclipse catalog's solar ϕ_{A3} (to check vs lunar ϕ_{A3}) or spot R's §C4 devolution.] The 1° discrepancy has been known at least since R.Newton 1977 p.119's clear explanation of the joke solar-speed which non-correction entails. (See also Hugh Thurston 2002S p.67. Duke 2005T p.176-177 n.5 ignores R.Newton, DR, & Thurston, though doing so results in a wild solar-orbit eccentricity[14] of over ***thirteen percent*** as 1st noted at Rawlins 1991W fn 162 [$e \doteq 8/60$], & [non-citationally] agreed to at Duke 2005T p.177 n.5.) But no one (incl. Rawlins 1991W) previously realized that the error was due to a deliberate shift — to forge an orbital fit. Frankensteinorbit (§E2) is obviously & variously superior (to a 13%-eccentric EH solar orbit!) — since its apogee-at-epoch $A_\circ$ (PH) and eccentricity[15] e (EH) are much nearer reality (e.g., eccentricity 5% vs then-actually 4%) than the 13%-eccentric monstrosity required if 1°-fudge isn't corrected-for. Concluding this section: We have established Hipparchos' adoption of a one-of-a-kind (and physically impossible) celestial configuration: a 179° difference of solar true longitude vs lunar true longitude for −381/12/12 mid-eclipse — adopted to paper over problems with his eclipse researches. In astronomers' terminology: a mid-eclipse Moon at 179° true elongation — Hipparchos' astonishing 1° fake, which has now (§G1) for the 1st time been fully solved, and thereby detected as fraud.

H Centrists & Rebels

H1 Several discussions of the Hipparchos trios have appeared in recent decades out of the history-of-astronomy center, e.g., Toomer 1973, Neugebauer 1975 pp.315-319, Jones 1991H, etc — along with two rebel studies, R.Newton 1977 p.115-129, and Rawlins 1991W. The former authors all take the data as entirely real; the latter dissent (in differing fashions).

H2 Contra Rawlins 1991W, centrist authors propose (or accept) that, for each trio, Hipparchos' analysis found unknowns *simultaneously* from his three time-data. It would indeed be possible thus to find three lunar-orbit unknowns; mean-longitude-at-epoch $\epsilon_\circ$ (in degrees), mean-anomaly-at-epoch $g_\circ$ (also in degrees); and eccentricity e (Trio A) or epicycle-radius r (Trio B), either expressed in 60ths of a unit. But the centrist studies instead attempt to apply a newly (and wholly) invented Hunt&Freeze technique to go beyond what used to be the limit for three equations of condition, to try pulling from the data four unknowns — adding R to the hunt. (Though in their determination to conjure e [or r] and R from the process, they neglect $g_\circ$ & $\epsilon_\circ$ — omissions we have not repeated: Rawlins 1991W eq.9 & here at §J & eq.8.) Comments: [a] All such studies forget the possibility that Hipparchos cites trios not because of his math but simply because, at any given place on Earth, lunar eclipses can naturally occur in threes, during a brief period: less than a year. (Though: see Rawlins 1996C fn 103 for two unusual tightquads: −831-830 & −720-719.) [b] Simple pair-analysis produces perfect hits (Rawlins 1991W §P2) on the Hipparchan e & r (327 2/3 & 247 1/2) sought for Trios A&B. (See arrowed data in §§G2&F2, resp.) Meanwhile — unless (§K1) the calculations are flagrantly fiddled to produce the desired result — none of the centrists' complex trio-math attempted solutions has ever reproduced either (much less both) of these same Hipparchan numbers.

H3 Nor has the untampered Toomer fallout-in-midmath (§I9) reverie hit upon either of Hipparchos' R values (3144 & 3122 1/2), while Rawlins 1991W's eqs.23&24 showed that one line (here at eq.5) of Aristarchos-based trig leads to both, on-the-nose: above, §§C2-C4.

H4 But hist.astron's political center insists (§A2; Rawlins 1991W §§D4&H2) upon preferring (some variant of) its own cult's 4 decade-old solution. No hist.astron-centrist scholar has ***ever*** (e.g., fn 22) let its readers in on the fact that all 4 of these numbers were (above §H2 [b] & §H3) *precisely* matched by *ibid*. Compare to the *history of science* center — which a decade ago courageously defied the *JHA*-Muffia cult by disseminating Rawlins 1991W's results in the world's leading history of science journal, *Isis* (Thurston 2002S). [In 2016, *Isis* lost 2002 Editor Margaret Rossiter's openness: see letter Rawlins 2018C.]

I Undead Fantasy: NonExistent 3438-Radius Greek Chord Table

I1 The impasse here is based on a fundamental Muffia misunderstanding that takes for granted ("virtually certain" [Duke 2005A p.5]) that Hipparchos' analyses of the 2 trios were by the elaborate method Ptolemy develops so ably at *Almajest* 4.6: from 3 eclipses' times, solve for 3 planar lunar orbital elements simultaneously, thus here the "Simultaneous Method" (§H2).

I2 Toomer 1973 claimed to have shown that Hipparchos used the Simultaneous Method, doing his trig by a chord table based upon circle-radius $R' = 3438$, which is the number of arcmin in a radian. Such tables were used the better part of a millennium later in India, which drew some astronomy from the Hellenistic tradition, thus the superficial [Greek army left India 325 BC, two centuries before Hipparchos] plausibility of the Muffia's refreshingly original idea. (An inspiration which must have greatly pleased Indian-astronomy specialist and fellow-BrownU scholar & mentor David Pingree.)

I3 But there is not-a-shred of direct evidence that Hipparchos ever used such an odd device as a 3438-based trig table. Thus, Toomeresque theorizing has managed to pioneer not-a-shredness-squared here vis-à-vis Greek astronomy: R being found on-the-fly (see below: §I9), plus an Indian trig table flourishing centuries before its earliest attestation. And the Muffia claims to detest speculation by outlanders.

I4 What drew Toomer into his theory was the crude proximity of 3438 to both Hipparchan lunar R values: 3144 & 3122 1/2. When he applied the Simultaneous Method to Hipparchos' data (3 distinct ways [§I9] for each trio): during the cascading stages & mergings of the attempted 3438-based computational reconstruction, there inevitably appeared somewhere (anywhere would do) numbers that roughly approximated these two Hipparchan elements — though none came convincingly close to actually matching them.

I5 The most immediately obvious clue that Toomer — though admirably masterful at the geometry involved — is pursuing a chimera is: if ratio 247 1/2 vs 3122 1/2 had not been based upon a start-out presumption of lunar orbit-radius 3122 1/2 (as shown above at eq.6) why would the ratio not be converted by Hipparchos to 99/1249, just like the conversion (below) of eq.19 to eq.18? (Even the same conversion-factor: 5/2.) This alone (and see analogously below at §§I11&K3) tells us that lunar-mean-distance radius 3122 1/2 was adopted BEFORE not DURING Hipparchos' calculation — a priority which is consistent with all known Greek astronomical work. Further indication (as noted at Rawlins 1991W

[14] Remember that what ancients (using adjusted circular orbits) called eccentricity was twice what moderns (using elliptical orbits) refer to by the same term. Rawlins 1991W fn 162 found that if eclipse A3's $-1°$ fudge is not accounted-for (i.e., undone), the data are consistent with $e = 7^{\mathrm{p}}46'$ or 12.9 percent ancient convention; 6.5 percent, modern.

[15] Again (fn 14): these e values are ancient-convention. Modern equivalents would be half as large.

fn 244): 3144 & 3122 1/2 *are within 1% of each other* — a central point ***never even noticed*** by those promoting disparate on-the-fly origins for these numbers, thereby implying the super-proximity is just an amazing accident, (§K3), instead of wondering whether it was symptomatic of a common origin, which has turned out to be the now-obvious truth (§C4).

I6 To effect his midstream-fallout version of the Simultaneous Method, Toomer follows the same calculation-procedure as *Almajest* 4.6. He starts by drawing a line from the Earth (point O in Toomer 1973's diagrams) through any one of the three eclipse positions (points $\mathrm{M_i}$ for i=1-to-3 or I-to-III, in his diagrams), calling B its other intersection-pt with the lunar orbit — and calling "s" the line-segment from O to B, a device that permits geometry to solve the problem. (In Duke 2005T's revisitation of Toomer 1973, s is called d.)

I7 Toomer adds to the speculativeness of his conception by supposing that Hipparchos did not use the Greek chord (crd) table of *Almajest* 1.11, where values range from 0 to 120 (angles 0°-to-180°):

$$\text{Greek:} \qquad \text{crd}\ \alpha = 120\sin(\alpha/2) \tag{12}$$

— but instead for each trig calculation took a number c.180/π times bigger:

$$\text{Indian:} \qquad \text{crd}\ \alpha = 6875\sin(\alpha/2) \tag{13}$$

I.e., Indian tables' values range between 0 and 6875, being based upon a circle-radius of 3438 units.[16] This ensures that each trig calculation is likely to produce numbers in the thousands. As each Toomer trio-calculation proceeds, one of these numbers of course may happen to hit near one of the two he's looking for (either will do). (If neither desideratum turns up, he can try the same calculation by drawing line s through either [§I6] of the other two eclipses of the trio and computing on that basis. [See Toomer 1973 p.27 n.14.] During each trio's Simultaneous Method analysis, $R' = 3438$ enters at step 3 due to the method's dropped-perpendicular ploy [*Alm* 4.6].) Let's watch it happen, starting with Trio A.

I8 Toomer 1973 pp.13-14 Trio A successive line-segments (units of s, except last line):

$$\frac{3110\ 1/2}{6574}$$

$$\frac{5379}{3247}$$

$$\frac{5379\cdot 6688}{3247\cdot 2\cdot 3438}$$

$$\frac{6268}{3438}$$

$$R = \frac{6268\cdot 3438}{3438\cdot 6688} = \frac{6268}{6688} = \frac{3134}{3344}\ \text{[why not} = \frac{1567}{1672}\text{? — see §I10]}$$

$$\frac{3134\cdot 6853}{3344\cdot 3438}$$

$$R/e = \frac{3134}{338}$$

(Exact computation all the way through would instead produce 3135/335 = 627/67.)

I9 Given that Toomer calculated his method from three different starting points (Toomer 1973 p.27 n.14), it is no great shock that somewhere along the way a number (3134 at §I8's step 5) indeed pops up that's near one of the Trio A sought-for R (3144) or e (327 2/3). The only genuine surprise here occurs when Toomer supposes that Hipparchos would at this point suddenly and selectively preserve the number 3134 all the way to the end of the calculation. (One can understand Toomer freezing such a number, since nearby 3144 is one of his goals. But why would Hipparchos care about packing this value in ice?)

[16] There being 21600 arcmin in a circle, the consistent radius is that number divided by 2π: slightly less than 3437 3/4, the number of arcmin in a radian. The Indian table proposed for Hipparchos (Toomer 1973 p.8, Duke 2005T p.175) is effectively the Ptolemy (*Almajest* 1.11) table at 7°1/2 intervals (see also ‡1 fn 1) — with each Ptolemy chord-value enhanced by factor 180/π and integrally rounded (4-place precision). See Neugebauer 1975 pp.299-300, 319, 1116, & p.1132 Table 8.

I10 The point becomes particularly relevant when we notice that the §I8 line (step 5) which produces $R = 3134/3344$ (in units of s): came from dividing numerator 6268 & denominator 6688 each by 2. But: why not divide by 2 again?! [Duke 2005T p.171 passes over same point when p.170 line 6 conjures-up 3144.] Why doesn't Toomer simplify 6268/6688 to 1567/1672 instead of 3134/3344? (Previous line: exact calculation produces 6270/3438, so equitably dividing by common factors 2&3 produces 1045/573.) Not remarking the plain divisibility of 6268 & 6688 by 4, Toomer 1973 p.27 n.14 just says (emph added): "I suppose division by the common factor 2 here. It must have occurred at some stage *in order to get Hipparchus' final result*." (Same circular reasoning below: §§I14&K1.) I.e., ***prove Hipparchos used the 3-unknown method by assuming he did***.

I11 The foregoing §I10 failure-to-divide-by-2 revelation is just another example of an earlier point (§I5) regarding Trio B: the very r/R ratio being sought, (247 1/2)/(3122 1/2), would have immediately (after factors' division by 5/2) become 99/1249. Similarly for Trio A's r/R ratio: (327 2/3)/3144 → 983/9432. Why carry fractions in a fraction unless something fundamental is being held fixed? So it makes more sense to suppose that Hipparchos had adopted his R values (3144 & 3122 1/2) before his mathematical searches for e & r even started.

I12 Toomer 1973 pp.10-11 Trio B successive line-segments (units of s, except last line):

$$\frac{1000}{6669\ 1/3}$$

$$\frac{1112\ 1/2}{6750\ 1/2}$$

$$\frac{2989\cdot 1112\ 1/2}{2\cdot 3438\cdot 6750\ 1/2} = \frac{246\ 1/3}{3438}$$

$$\frac{5\ 1/3}{3438}$$

$$\frac{246\ 1/3}{3438}$$

$$r = \frac{3438\cdot 246\ 1/3}{2989\cdot 3438} = \frac{246\ 1/3}{2989}$$

$$\frac{2372\cdot 246\ 1/3}{2989\cdot 3438}$$

$$R/r = \frac{2913}{246\ 1/3}$$

I13 When 246 1/3 (close to Hipparchos' $r = 247$ 1/2) appears (in §I12's step 3), it is not multiplied or divided in the steps that follow, thereby ensuring its suspended-animation-survival for ultimate display in a ratio for comparison to Hipparchos'. So the near-grail of 246 1/3 is captured only due to the *choice* to set aside and retain 3438 in the 3rd line of the above, while simultaneously *merging* four other numbers:

$$\frac{2989\cdot 1112\ 1/2}{2\cdot 6750\ 1/2} = 246\ 1/3 \tag{14}$$

(Exact calculation from the outset yields 2990 & $r = 246$ 1/2, and $R = 2918$ or 3079; the former from a non-fudged diagram; the latter, from allowing Toomer's below [§I14] re-draw of it.) Toomer 1973 p.11 realizes that 2913 isn't near the desired $R = 3122$ 1/2. So, does this show that that the theory is disconfirmed? No, it continues (Rawlins 1991W §P1) in high regard at hist.astron's political center.

I14 Undeterred by 2913's remoteness from 3122, Toomer&co have attempted to FORCE this by-now Muffia-sacred method to work. Toomer 1973 was first to do so, proposing (p.11) a weird error of method. (Which still failed to find good matches.) His reasoning: if the number doesn't match, "we must . . . suppose that [Hipparchus] made a mis-calculation."

Toomer 1973 pp.11-12 reshuffles his diagram (admittedly falsely, supposing that Hipparchos made the same step inadvertently), and this time he gets (3082 2/3)/(246 1/3). Remarkably, considering Toomer has still missed by c.40 units a mark (3122 1/2) which Rawlins 1991W eq.24 was to hit *infinitely* more closely (above eq.6), Toomer concludes (emph added): "This is sufficiently close to Hipparchos' ratio [R/r = (3122 1/2)/(247 1/2)] to ***prove*** that Hipparchus did indeed use a chord table of the type [here] posited in computing it".

I15 A few years later, the foregoing Trio B development collapsed when an input datum was found (Toomer 1984 p.215 n.75) to be based upon a false reading of mss. (Computation using the correct reading led to an r of about 231 [Duke 2005T gets 231.727: see below at §K6] instead of 246 1/3. So Toomer's ***proof*** was a fantasy all along: *the imagined match was purely coincidental*.) To his credit, Toomer (at least temporarily) agreed that this correction "cast doubt" (*loc cit*) on his claim Hipparchos used a 3438-based chord table.

J Hipparchos' Overconsistent $g_\circ$ Reveals Aristarchos' $A_\circ$ = 96°

No one pushing Hipparchos' use of Ptolemy's Simultaneous Method (3 unknowns: §I1) ***uses it himself*** to find mean-anomaly-at-epoch $g_\circ$. *As Ptolemy always did* (*Almajest* 4.6), though *Alm* 4.11 cites Hipparchos' solution for **merely 1 unknown** for each trio, comparing his own lunar e (or r) to Hipparchos' but ***making no comparisons*** for the other 2 orbital data his 3-unknown method could find: $g_\circ$ & $\epsilon_\circ$. (Telling all but Muffiosi that Hipparchos neither sought nor cited either.) Three-unknown-solving for $g_\circ$ led DR to the entire trios-mystery's solution (Rawlins 1991W §§N4&N10): Trio A's $g_\circ \doteq 81°.8$; Trio B's, 82°.6. While quite inaccurate (real $g_\circ \doteq 87°$; *Alm*'s, 85°.3), the $g_\circ$ are near-equal (fn 9), shockingly so, given [i] $g_\circ$'s sensitivity to input-data uncertainty, & [ii] the big disparities of Trio A's e vs Trio B's r, and of the trio-analyses' underlying solar apogees (21° apart! *ibid* §K9vs§M4). These clues plus Rawlins 1991W §N10 are what suggested (*ibid* §§N4-N11) that $g_\circ$ was not sought by Hipparchos' math but rather was from a predecessor and thus set at 82°, so $A_\circ = 96°$ (*ibid* §N5 & eq.9; above eq.8) for both trios from the start. *Ibid* §§N4&N17 assign these (poor) values to Aristarchos. But Trio B's date could indicate lunar specialist Apollonios.

K Mythic-Centaurus Refereeing OKs Riggerous Mathematical Proof

K1 Nearly a third of a century after Toomer 1973, the journal *Centaurus*[17] published Duke 2005T, which attempted to salvage Toomer's theory by (for each trio, at a chosen step) changing one number to (***unlike above eq.5→eq.6***) an unrelated number, to MAKE said theory fit. For Trio A, presuming Hipparchos mis-computed ζ_3 (our $g_{A1} - g_{A3} + \delta_{A1\text{-}A3}$) as $51°19'37''$ (though accurate Duke 2005T calculation would yield $51°30'23''$) — openly stating no other justification but that this was necessary to get the Right Answer: Duke 2005T p.170. (Similar to Toomer: above at §I10.) Quoting *idem* on Trio A (emph added): "In order to get Hipparchus' answer we have to invoke some amount of rounding and miscalculation, so the first step is *to adjust something so that the correct numerical value* for the ratio R/e [3144/(327 2/3)] is produced. One simple way to accomplish this, out of an infinitude[18] of choices, is to assume that Hipparchus miscomputed [51;30,23 of *idem* p.169] as 51;19,37, but did everything else precisely. Then he would get" the Right

[17] Even beyond *Centaurus*' imperviousness to the (creditably undisguised) *ad-hok*iness of the proposed processes (esp. Trio B) — which led to *DIO* referee Hugh Thurston's rejection of them — there are printing problems here (none of which affect Duke's uniformly accurate calculations). These again (as we saw at R.Newton 1991 fn 7, Rawlins 1991W fn 126, Rawlins 1996C §B6) reveal hollow refereeing at *Centaurus*: [a] Nest of misprints in p.168's last paragraph (e.g., for $\alpha_3 = 360° - \alpha_1$ read $\alpha_3 = 360° - \alpha_1 - \alpha_2$). [b] At p.169 line 5, sign-typo; line 6, read 3135 1/7 for 3155 1/7. [c] Sign-slips in formula for R, at pp.172&173. (Harmless: Duke uses correct sign in actual calculations.)

[18] *Ibid* (pp.171&173) sees firm links between 3144&3438 and 247 1/2&3162; but each supposed link depends upon a specific choice of data-alteration among the cited infinitude of other possible options.

Answer[19] by *Alm* 4.6's method, there being "no question" (Duke 2008W p.286) he knew it.

K2 The one seeming "hit" here (presumably creating a sense of progress) is this: while Toomer 1973 p.15's 3134/338 got neither the Hipparchan e/R ratio (327 2/3 vs 3144) *nor* either of its factors, Duke 2005T p.170 by contrast nearly achieves all *simultaneously* by discerning (§K1) that changing only ζ_3 and by JUST (fn 19) the Right amount produces 3144 3/8 and 327 5/7, where the latter figure is taken as roundable to 327 2/3. (Would've been rounded to 327 3/4? Problemlet fixed at fn 19.) Such a match would lead any curious, able scholar onward to follow-up-investigate what initially smells like a faint hope of success. But such simultaneity's failure (§K4 below; Duke 2005T p.172) for Trio B suggests that Trio A's match was just coincidence, a likelihood enhanced by the admittedly (§K1) huge range of options by which one may manipulate the Trio B numbers until finally (eq.15) getting lucky. And enhanced further by realization that an utterly different explanation (Rawlins 1991W) solves both trios exactly. Note the stark essential contrast between Duke 2005T and the present analysis: all DR solutions involve ***ancient-typically round numbers, are independently***[20] ***supported*** and (unlike Duke 2005T p.172) ***equitably consistent for both trios*** (even the very same $g_\circ$ & $\epsilon_\circ$), ***while Occamly forsaking adducement of convenient Hipparchan miscomputations***. Summarizing: Aristarchos' famous 87° recovers $R = 3144$ (eq.5), which becomes 3122 1/2 (eq.6) via notoriously common misread (§C4); element-borrowing revealed by ultraclose $g_\circ$-agreement with 82° (§J; *and see especially* Rawlins 1991W §N10) for otherwise-jarring Trios A&B, **thereby ruling-out the 3-unknown Simultaneous Method**, which seeks $g_\circ$ as an unknown.

K3 An extra problem with the entire initial presumption that Trio A's 3144 & 327 2/3 materialized during a long mathematical juggle: were 3144 not fixed as R from the start, it would disappear through an obvious simplification; in a problem whose shaky input data (& discrepant e&r) would have suggested relative uncertainties of ordmag 10%, it would affect ratio R/r by barely 1 part in 1000 (ordmag 1/100$^{\text{th}}$ of ratio's uncertainty) to just round e to 328, then remove the factor 8 from numerator & denominator, leaving the neat ratio 393/41. This point reminds us again (as at §I5) that the proposed Toomeresque-processes for the two trios involve (§I10) so many arbitrary choices and cancellations, that it would be remarkable if the two R values agreed to within 1% by the accident implicitly proposed.

K4 For Trio B, a plainly unrelated number is again substituted for another (8;44,08 for Duke 2005T's $\delta_{B3\text{-}B2}$ = 8;46,28), again of just the right size that ratio r/R comes out equal to Hipparchos'. But even this ploy won't produce Hipparchos' absolute values for r & R, as had been barely (fn 19) possible for Trio A; so, after this Trio B analysis gets as far as fixing the Right Ratio, a boldly arbitrary, Occam-defying *d-ex-machina* is brought in. A mere passing computation-facilitator (properly playing no rôle in final element-values of Ptolemy, Toomer 1973, or Duke 2005T's Trio A, as noted: p.172), final-fiddle-factor d is now for Trio B *ad-hoc*-manipulated by just arbitarily setting it at whatever (unattested) value neatly converts §K4-adjusted r = 231 3/4 into 247 1/2 (p.172 step 5): namely, 3162.

K5 This also forces R to come out 3122 1/2 (step 8), since fudgery of $\delta_{B3\text{-}B2}$ (at §K4) had already been precisely designed to guarantee the process' issuance of the Right Ratio (12.616, as found unmanipulatively at Rawlins 1991W §N14); which, if simply multiplied times r = 247 1/2, produces the desired attested (eq.4) R = 3122 1/2. The astonishing[21]

[19] Beyond the 51° integral part, there's no resemblance between the former & latter (supposedly miscomputed) digits. (Thus the hemmed-in precision of the choice of substitute number is revealing: the only values that would produce a ratio roundable to 3144:327 2/3 are between 51;19,32 & 51;19,41. Actually, 50;19,33-36 would do better than Duke 2005T p.172's 50;19,37, since r would be more convincingly roundable to 327 2/3 than is 327.719 [likely to be rounded to 327 3/4], which follows from 50;19,37.)

[20] E.g., Rawlins 1991W §§K4&L3; *DIO 6* ‡3 §D6.

[21] We are asked to accept that Hipparchos' Trio B analysis specially used a π-value good to barely 2 decimal places — way worse than any known in competent antiquity (22/7, 377/120, Duke 2005T n.3, etc) — even in the very midst of the §I12 calculations, all of which are done to at least 4-place precision, **necessarily** using trig tables of like precision. See Toomer 1984 p.57 [n.68]; and note that

adducement of 3162 (yet-another goal-directed special assumption) is justified by some way-later (obviously not precision-addicted) Indian astrologers' use of $\sqrt{10} = 3.162$ for π. (Unheard-of for Greek astronomy; & *extant Indian records nowhere exhibit* $R' = 3162$.)

K6 In the step that produces r (§I12 step 3), instead of following Toomer's arbitrarily freezing the numerator (231.727, after §I15's correction) it's instead *differently*-arbitrarily treated: merged with its denominator (c.2960) and the result multiplied by $d = 3162$:

$$r = 1000 \cdot \sqrt{10} \cdot 231.727/2960.39 = 247.53 \doteq 247\ 1/2 \tag{15}$$

The equation for R (Toomer 1984 p.11) then nearly becomes (Duke 2005T p.173):

$$R = \sqrt{3162^2 + [247\ 1/2]^2 - 3162 \cdot 106\ 1/7} \doteq 3122\ 1/2 \tag{16}$$

But eq.16 doesn't work as printed (at *ibid* p.173): 106 1/7 is a harmless remnant resulting from one of a succession of shopping-trips[22] seeking a way of fitting the calculation to Hipparchos' results. (Back when $d = 3438$ was tried-out before going for 3162.) Once we switch fully over to $d = 3162$, the intended equation (which Duke actually&correctly computed with) appears [though exact math all the way yields c.3122 2/3]:

$$R = \sqrt{3162^2 + [247\ 1/2]^2 - 3162 \cdot 97\ 2/3} \doteq 3122\ 1/2 \tag{17}$$

L Hipparchos in Toto

L1 *DIO* has defied 100^{y} of Experts' mis-ascribing to Babylon Hipparchos' amazingly accurate draconitic period-relation:

$$5458^{\text{u}} = 5923^{\text{w}} \quad [= 5849^{\text{v}} + 147°] \tag{18}$$

(Where we've appended in brackets the seemingly hopeless anomalistic situation, for contrast with near half-integrality below at eq.19's ***precise*** resolution of the mystery of eq.18's origin. Superscripts: u = synodic months, v = anomalistic months, w = draconitic months.) Said consensus asserts that Babylonians used this equation c.200BC (long before Hipparchos), citing 6 cuneiform-text lunar data (Rawlins 2002H §D1) calculated for that time.

L2 But (as noted at *idem*) there is a striking and **perfect** correlation here (which reminds us that ephemerides FOR a given year are frequently computed AT a quite different year): three of the six texts do indeed use eq.18 ***but they are not dated on the clay*** — while all three of the texts which ARE dated on-the-clay (dated as created c.200 BC) do NOT use eq.18. Ah, but there is a 7th text which is clay-dated — and uses eq.18. So that clinches it for the centrists? No — ITS date is well after Hipparchos. (Full discussion: Rawlins 2002H §D.) So the supposed impediment (to accepting *Almajest* 4.2&9's attribution of eq.18 to Hipparchos) actually just adds — 7-times-out-of-7 — to the pro-Hipparchos evidences. These evidences were already manifold, precise, and in some cases jaw-dropping.

E.g., eq.18 can only be based on the standard Greek method of finding lunar motion (namely, eclipse-cycles) if an ***apogee-perigee*** eclipse-pair was used (Rawlins 2002H §B):

$$13645^{\text{u}} = 14807^{\text{w}}1/2 = 14623^{\text{v}}1/2 + 7° \tag{19}$$

L3 Division by 5/2 (like §I5) produced eq.18. Clincher: the only eclipse-analyst known to make the highly peculiar choice of apogee-perigee pair is Hipparchos. And he is attested (*Almajest* 6.9) as doing so using the very −140/1/27 perigee lunar eclipse which Rawlins 2002H showed would ***precisely account for all eight digits*** in eq.18 if Hipparchos compared it to a (since-lost) record of −1244/11/13's apogee eclipse (instead of comparing it to the −719/3/8-9 apogee eclipse he used for his 1st try: *Almajest* 6.9). We know period-relation eq.18 is not from predecessors (as at §J) since he saw the −140 eclipse (*Almajest* 6.5&9). Rawlins 2002H §C provides a detailed survey of the ***SIX-FOLD*** array[23] of such testimonial, methodological, & quantitative verifications of Hipparchos' authorship of eq.18.

L4 Discovery of precession is commonly mis-attributed to Hipparchos, though it was undeniably (Rawlins 1999) known to Aristarchos over a century earlier. So eq.18 is easily Hipparchos' greatest scientific discovery. Rather than subtle math, finding eq.18 primarily required dedicated determination: laboriously filtering extremely ancient records (over 1000^{y} old to him — obviously part of "the series[24] brought over from Babylon": *Almajest* 4.11) added to Aristarchos-level (www.dioi.org/cot.htm#tqdr) fine judgement in eclipse-choice. The result was (& *is*) accurate to 1 part in ordmag *ten million*. And not by accident. So, when considering Hipparchos' lesser moments (due to the math limitations of himself & his colleagues, especially early on), we should keep in mind his marvelous eq.19→eq.18 advance of lunar theory,[25] a decade-old *DIO* discovery (Rawlins 2002H) still nowhere even understood (much less accepted) by hist.astron archondum, which gave up fighting us after instant-torpedo-reversal (§L2) produced only the dreary longterm downer[26]

$R' = 3438$ is found by dividing a 4-place-accurate value for π into 10800.

[22] Another symptom of the flexibility of the search for a path to a fit: *Centaurus* also missed two places where a choice for chord-table-radius R' was in-flux — thus left for later filling-in, but never got filled. At p.164, we read that Toomer's "hypothesis that Hipparchus had used a chord table of radius was correct". And at p.172: "following the same path as in Trio A using does not give". The omissions are again harmless, but together they show that not only did *Centaurus* check none of the math, it didn't even read the text. Worst: *Centaurus* didn't require citation of Rawlins 1991W, and-or its prominent summation by Thurston 2002S (nor did *JHA* for the Babylonianist-Muffiose paper Duke 2008W pp.290, 293, 294 & nn.22-23; on n.24, see *DIO 6* ‡3 §§D&H2!!), which had ***historically*** (*ibid* §D6) recovered all 4 Hipparchos numbers exactly; this, even though Duke 2005T **was born to bury Rawlins 1991W**. (Duke 2005T started a decade ago as an entry for *DIO*'s van der Waerden Award, which invites [www.dioi.org/pri.htm] such challenges.) But, without Duke 2005T's stimulus (plus Toomer 1973's & Jones 1991H's, earlier), DR probably wouldn't have ever discovered this paper's most startling new result (§G1), neatly consistent with the §E7 Pair Method hypothesis: that Hipparchos' school had — due to problems with said method — clumsily (Rawlins 1991W §N15) faked a 179° true-elongation lunar mid-eclipse. That the legendary "Father of Astronomy" was involved in fraud merely adds to the list of eminent figures whom *DIO* has investigated in connexion with such activity. E.g., Ptolemy (Pickering 2002A, Duke 2002C), Tycho Brahe (Rawlins 1992T), Marlowe-Shakespeare (www.dioi.org/sha.htm), A.Robertson (Standish 1997), Isaac Hayes (www.dioi.org/hay.htm), R.Peary (Christiansen 1997), R.Byrd (www.dioi.org/byf.htm). [Also: fake-refereeing at some extremely handsome journals, e.g., *Centaurus* (just above; or fn 17) and Lord Hoskin's shunloving (Rawlins 1991W §B3) *Journal for the History of Astronomy* (www.dioi.org/jha.htm#kqlz).] But most of these figures left more positive than negative legacies. Is there a correlation between [a] exaggerations too often attendant to fundraising needed for great deeds, & [b] sham that also-too-often attends moments of falling short of greatness?

[23] Impervious to every item of this devastating series of childishly obvious indicia, our ever-ineducable *DIO*-denier klan instantly hurled its six-texts killer-wannabe torpedo (§L1) at *DIO*. When that shot just-as-instantly backfired (§L2), *DIO* was hugely & at-the-time-gratefully (Rawlins 2002H §D1) enlightened. But, again: Muffiosi learned nothing — and just skulkingly departed discussion *without a word on their latest ideakiller-dud*. It's so efficient&comforting — and so like C.Ptolemy — to know answers ahead of incoming evidences, regardless of blow after blow after blow after blow of such.

[24] The early eclipse records from Babylon may have been pretty dense at least in patches. How else explain that Hipparchos was able to search through and find a just-right match to establish eq.18?

[25] J.C.Adams' 1846 Neptune fiasco (Rawlins 1999N) was followed by redemption through his brilliant, ultimately fruitful pioneering discovery (Rawlins 1992W §I12) of lunar theory's discord with observation, due to Earth-spin acceleration, the prime research field of the eminent Johns Hopkins physicist R.Newton. This work eventually led him to expose Ptolemy's fraudulence (e.g., R.Newton 1977), triggering decades of ethically repulsive AmerAstronSoc-HAD-*JHA*-Muffia denigrations, non-citations, suppressions, & shunning of him, an academic obscenity now surviving by transference of target to DR. None of which appears to upset any of those vaunted watchdogs we keep hearing about, whenever archons try to convince Congress that academic misbehavior is of trifling dimensions.

[26] But the frustration-downer for seething semi-numerate archons is matched by a kinda-upper for *DIO*. We prefer & repeatedly invite communication and-or (www.dioi.org/deb.htm) debate even with the doltiest of the hist.astron field's archons. But, failing that, it is tragicomically entertaining to watch their political obsessions retard their own field's progress by barring valid scholarship — thereby (as we predicted two decades ago: Rawlins 1991W §P3) "betraying their very profession. Not every scholar's detractors are so obligingly cooperative in thus destroying their own intrinsic credibility."

of finding no *DIO* math error, alternate eclipses, or equally accurate methods that *could match DIO by eliciting all eight of Hipparchos' digits*. Exactly.[27] Again: née-jerk archons yet lack the sense&balance to gauge, face, or admit the obvious uniqueness of our solution to one of the key equations in all ancient astronomy. In other words: the usual.

L5 We remember also outdoor-Hipparchos' establishment of the earliest extant grand star catalog, a treasure stolen by indoor-Ptolemy, but lately restored to its rightful creator, through researches (R.Newton 1977, Rawlins 1982C, Graßhoff 1990, Rawlins 1994L, Pickering 2002A, Duke 2002C) none of which appeared in hist.astron's centrist & "premier" journals, who instead spat on the truth for decades (1975-2002), to the extent of scores of pages of reliably humorous pseudo-scholarship. (Highly admirable Muffia exception: Toomer's swift conversion by and publication of epochal Graßhoff 1990 via Springer.)

L6 The present article and the previous are our latest installments in *DIO*'s ongoing (from *DIO 1.1* to date) expanded view of Hipparchos' evolution from amateur (c.−160) into a serious contributor (c.−130) to the growth of astronomy. Our journey has been, like his and his science's, a Niagara of surprises: the irresistible lure of the inductive journey.

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[27] This holds for all three of *DIO*'s solutions of the origins of anciently-adopted lunar speed estimates: ***twenty-four digits*** in all, each reproduced exactly. Details: www.dioi.org/thr.htm & *DIO 16* p.2.