‡6 Greek Accuracy-Secret: Integral Period-Ratios

Monthlengths Known to 1^s Accuracy — 2000^y Ago

A Three Accurate Monthlengths

A1 In the late BC era, celestial bodies' speeds were (e.g., O.Neugebauer *History of Ancient Mathematical Astronomy* [*HAMA*] 1975 pp.69 and p.906 Table 15; www.dioi.org/jb12 §H) expressed as

RATIOS-of-INTEGERS.

Historians-of-science have been excruciatingly loathe to admit the obvious, namely, that this was because said speeds were discovered by observing, of all things:

RATIOS-of-INTEGERS.

A2 On this point, orthodoxy, long committed to the position that ancient astronomers were inaccurate bumblers or even fabricators, has generally, slavishly (*DIO 22* ‡2 §N8) believed "The Greatest Astronomy of Antiquity" (so headline-deemed[1] in AAAS' *Science 193*:476 [1976/8/6]) who pretends (*Almajest* 4.6-7, 9.10-11, etc) that celestial speeds were found from long nonintegral revolutions between observations, divided by corresponding time intervals. (Yet even Ptolemy realizes [*Almajest* 4.6] that the best estimates of celestial bodies' motions were derived from data taken over vast intervals of time, a key foundation for what follows here.)

A3 Below, using the integer-ratio technique, we will recover the **EXACT** empirical bases explaining how ***all 3 ancient monthlengths — synodic month, anomalistic month, & draconitic month — were, by the 2^{nd} century BC, determined to 1^s accuracy or better, without the aid of telescopes or reliable clocks***.

B The Integral Method

B1 How?! Simpler than the astonishing results would suggest. *Almajest* 4.2 (2^{nd} century AD) says ancients had spotlighted an eclipse cycle nearly 345^y long: 4267 synodic months. (The synodic month is the $29^d1/2$ civil month we are familiar with.) This period agreed almost exactly with 4573 anomalistic months. (The anomalistic month is the mean time the Moon takes from perigee back to perigee.) So the interval between pairs of eclipses for this ratio was virtually constant, no matter where on the ecliptic the eclipse-pair occurred, or when — because whatever the 1^{st} eclipse's orbital-position irregularity (due to eccentric motion), it was almost *precisely* matched by the 2^{nd}'s irregularity and so the two effects cancelled. Centuries of Babylonian eclipse data preserved at Babylon, and later "brought over" (*Almajest* 4.11) to Alexandria, compared to like data observed in classical antiquity, showed that (due to such cancellations) ***any*** 4267 month eclipse-pair interval **never varied** — regardless of zodiacal positions — by more than a fraction of an hour from a mean of 126007^d01^h (*Almajest* 4.2). Dividing that interval AND its 1^h- error by 4267 yielded $29^d12^h44^m03^s.3$, **accurate to better than 1 time-second** (i.e., $1^h/4267 = 3600^s/4267 < 1^s$).

[1] See also *Science 199*:872 (1978/2/24) & History-of-science Society's *Isis 93*:70 (2002 March).

B2 Sure enough, we find a monthlength of $29^d12^h44^m03^s1/3$ explicitly attested on cuneiform text BM55555 (c.100 BC). Also at *Almajest* 4.2 (c.160 AD), which says: [1] that this was the monthlength of Hipparchos (c.130 BC), and [2] that the 223 month saros expression, which has been (T.Heath *Aristarchus of Samos* 1913 pp.314f; *DIO 11.1* ‡1) mathematically traced to Aristarchos (280 BC), was $18^y10°2/3$, one 223[rd] of which is $29^d12^h44^m03^s.23$, agreeing to 1 part in 24 million with the "Babylonian" value, and to 1 part in 76 million with that value's actual empirical basis ($126007^d01^h/4267$: §B1 above), that is: **to within 1/30[th] of a time-second**. Both micro-differences are just typically negligible rounding-imprecisions, so we may take all three as effectively identical. This ***Aristarchan*** monthlength, later (c.200 BC) adopted by Babylon (and today commonly miscalled the "Babylonian" or "System B" month) and even later by Hipparchos, also agreed with reality — to a fraction of a time-sec (as predicted above at §B1, from the 4267 month interval's constancy), since the synodic month was then $29^d12^h44^m03^s1/2$.

B3 Since 4267 synodic months is 4573 anomalistic months, removing (*Almajest* 4.2) the shared factor 17 yielded the standard ancient equation: 251 synodic months = 269 anomalistic months. Which made it easy for ancients to determine the anomalistic month by simply multiplying 251/269 times §B2's synodic month, yielding: $27^d13^h18^m35^s$, just 1^s less than the truth then.

B4 The draconitic month — or "eclipse month" — is the mean time for the Moon's return to a node, where eclipses can occur. The method of determining its length is provided at *Almajest* 6.9, where Hipparchos preliminarily chooses an appropriate eclipse-pair of similar magnitudes: Babylon-observed 720 BC March 8 and Hipparchos-observed 141 BC January 27 separated by almost exactly 7160 synodic months and 7770 draconitic months, so the eclipse month would be 716/777 times the synodic month, or $27^d05^h05^m37^s.0$, too high by 1^s. This is not the ratio he finally concluded for, namely, 5458/5923 (*Almajest* 4.2), for which no one (before the 21[st] century) had found the right eclipse-pair solution, since all feared to search back before the orthodox 721 BC limit (for Babylonian eclipses available to the Greeks), looking for the earlier eclipse that would satisfactorily pair with a classical-era eclipse, thus solving (by identical proportion) the attested 5458/5923 period-relation. In 2002, it was finally realized (*DIO 11.1* ‡3 §B) that Hipparchos, seeing that a longer interval would improve accuracy, switched (*idem*) his choice of prior eclipse back to a now-lost Babylonian record of the 1245 BC Nov 13 eclipse, while retaining the same attested original first-hand observation he'd already used: 141 BC Jan 27. This established that 13645 synodic months equalled 14807 1/2 draconitic months, which after division by 5/2 produced the ratio 5458 synodic months = 5923 draconitic months, attested at *Almajest* 4.2.

B5 Next, Hipparchos could just multiply §B1's synodic month by 5458/5923, to find that the draconitic month = $27^d05^h05^m35^s.9$, off by but a fraction of 1^s, the actual value then being $27^d05^h05^m36^s$.

C How Low Integral Numbers Can Produce High Precision

C1 When non-mathematicians 1[st] hear that 355/113 equals π to better than 1 part in a million, they wonder: how can such small numbers *do that?* The answer is that there are so many possible ratios to choose from (over 100000 pairs of integers less than 355) that the existence of a quantity's high-precision matching ratio is assured (and is readily found by continued-fraction analysis).

C2 The same logic applies to the plenitude of eclipse-pair-choice options that are available over an interval which extends *centuries*, guaranteeing the existence of accurate ratios. But there are not many satisfactory eclipse-pairs that are visible (both umbral & above-horizon) and whose latter eclipse is reasonably close to the ratio's approximate discovery-date.

D Summarizing Foregoing Empirical Bases of Greek Monthlengths

[1] 4267 synodic months = 4573 anomalistic months = 126007^d01^h
[2] 13645 synodic months = 14807 1/2 draconitic months
thus, dividing by 17 and 2 1/2, respectively:
[1] 251 synodic months = 269 anomalistic months
[2] 5458 synodic months = 5923 draconitic months.

E Three New Solutions

E1 Prior to 2002-2003, three lunisolar-speed ratio-equations lacked solution:
[A] 6247 synodic months = 6695 anomalistic months (System A 263 BC);
[B] 5458 synodic months = 5923 draconitic months (Hipparchos 141 BC) [already listed];
[C] 3277 synodic months = 3512 anomalistic months (Ptolemy 136 AD).

E2 The specific pairs that satisfy the three attested ratios (and the obvious strictures of §C2) are listed in *DIO 11* ‡2 & ‡3, *DIO 13* ‡2, and *DIO 22* ‡3 §I36. The best candidates (leading directly to ratios [A]&[B]&[C] of above §E1):
[A] 1292 BC Nov 23 vs 281 BC Jan 16 & 1274 BC Dec 05 vs 263 BC Jan 26
[B] 1245 BC Nov 13 vs 141 BC Jan 27
[C] 1201 BC Jul 11 vs 125 AD Apr 05 & 1190 BC Jun 12 vs 136 AD Mar 06
Though the latter eclipses extend over nearly 4 centuries (263 BC to 136 AD), each solution's early eclipse is from around the 13[th] century BC, a 4-times-narrower range — quite a remarkable circumstance, given that two independent indicia (www.dioi.org/thr.htm#rbkv and www.dioi.org/thr.htm#bsvx) also point to the vicinity of that epoch as the birth of Babylonian empirical astronomy.

E3 The mute history-of-science commune doesn't accept any of the above, though it has never even claimed that it has ***ACTUALLY SOLVED*** even one of the 3 relations we've here elucidated (§E2), and it cannot deny any part of the math which *DIO* has wielded to accomplish just that (nor deny the eclipses were above-horizon). In order to render undeniable the inductive asymmetry between the two camps: *DIO* has since 2018 sent to 1000 prominent scholars our offer, until 2020/1/1, of $100,000 (rules specified: *DIO 22* ‡3 §I38) in awards for alternate eclipse-pairs (starting no earlier than orthodoxy's 721 BC cutoff date) proving the non-uniqueness of *DIO*'s solutions. No such alternate eclipse-pairs have been forthcoming, thus the *DIO* eclipse-pairs (§E2, above) stand alone as those which have for the 1[st] time solved all three hitherto mysterious ancient ratios — that is, ***all the 24 digits*** of §E1, on-the-nose in every case.

E4 The Greeks' possession & use of Babylonian eclipse records from this early a period has never previously been broached, but the impressive accuracy of the monthlengths themselves argue in favor of great remoteness, for only through dividing endpoints' error by a huge number (§B1) could such accurate results have been attained — an achievement that will endure as a magnificent mutual monument to Babylonian scrupulousness and Greek ingenuity.

E5 A Debt to Ptolemy (‡8 §E1):
[1] If it weren't for the *Almajest*'s testimony on ancient method, we'd probably be seeing bestsellers claiming that such accurate monthlengths were brought to Greece via Meso-Americans, the Dogon, or alien visitors.
[2] As for §E3's $100,000 wager: in all the months since it was distributed (2018 Summer in *DIO 22*), not a cent has even been claimed. So, to anyone outside the History-of-science cult, it will seem a reasonable conclusion that *DIO*'s exact solutions (*idem*) to the 3 previously unsolved lunar-speed ratio-equations are historically actual.