The Acropolis Width and Ancient Geodesy

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"Was the Earth measured in remote antiquity?" This was the stirring question with which Berriman opened his book, *Ancient Metrology.* [1] To be sure, the question had earlier been tackled in Nicholson's *Men and Measures,* [2] as to whether a knowledge of Earth's dimension had afforded the original basis for units of measure. Here we enquire, specifically, as to whether the ancient Greek units of measure were related to the circumference of the Earth. This hypothesis tends to be related to the notion that a global, maritime civilization had once existed in prehistory. Our enquiry is therefore in some degree related to the thesis propounded by Francis Bacon, in his *New Atlantis*: "You shall understand (that which you will scarce think credible) that about three thousand years ago, or somewhat more, the navigation of the world (especially for remote voyages) was greater than at this day." [3] Could there have been a civilization of prehistory which vanished, but left behind its geodetically-defined units?

By way of investigating this, the dimensions of just two ancient buildings will be evaluated, namely the Parthenon in Greece and Stonehenge in England, taking only one measure from each. Bacon's statement concerning a now-forgotten early maritime civilisation will be taken as implying, were it true, an ability to measure longitude; and that in turn will be taken to imply some estimate of the Earth's circumference. It is evident that we cannot proceed along this line of enquiry without calling into question the most central assumption concerning the historiography of modern science as it has developed since the nineteenth century, namely that of a gradual increase of knowledge from a condition of primal ignorance. Should truth be found in the line of enquiry we are pursuing, then the story according to which the concept of progress is canonically articulated, will have to be rejected; and instead we will in some degree be back to the notion of a *priscia sapientia* as was believed in by Newton and Bacon.

The position here advocated could be described as Newtonian, insofar as Isaac Newton believed, that one function of ancient temples was that of expressing the proportions of the world:

> *"So then twas one designe of ye first institution of ye true religion to propose to mankind by ye frame of ye ancient Temples, the study of the frame of the world as the true Temple of ye living great God they worshipped. And thence it was yt ye Priests anciently were above other men well skilled in ye knowledge of ye true frame of Nature."*[4]

Our question will revolve around the Greek and Roman lengths of the foot, and also the longer unit of the "stade" - the original length of a racetrack. No comment will be made about the Great Pyramid, for which the reader's gratitude is expected, and Thom's "Megalithic Yard" will only receive a passing mention.

Precision of the Parthenon

The Acropolis, crown of the Parthenon in Athens and temple to Minerva, goddess of wisdom and learning, was also known as the *Hecatompedon*, a word that means "measure of a hundred feet." Thus Francis Penrose, an architect and scholar of the history of architecture, wrote: "There can be little doubt but that the front of the temple, which was always accessible for reference as a standard, was the true *Hecatompedon* in point of exact measurement."[5]

The temple was constructed to be 225 feet long and 100 feet wide. In 1882, Penrose carefully ascertained this length and breadth, "measured on its upper step," and thereby estimated the Greek foot to have been between 12.160 and 12.167 inches. He added rather casually, in a footnote, "The breadth, 101.34 is exactly a second of latitude at the equator."[6] How exact was that?

Penrose' statement is exact to around one part per thousand, which isn't bad, however we can do better than that. Taking the mean value of 100 Greek feet, as given by both the width and length of the Acropolis, and taking a mean value of the Earth's radius, [7] one thereby gains a tie-up of *three parts in 100,000*. That is around the limit of accuracy of the measurements involved. In other words, only by using the most modern values of Earth's mean radius, are we able to calibrate the precision with which the Acropolis was constructed, twenty-five centuries ago. Using the Earth's equatorial radius gives too large a value,

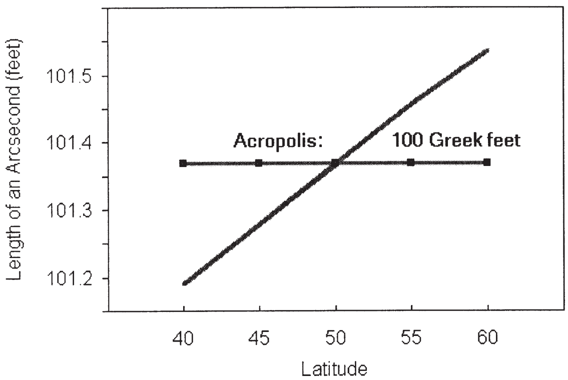

Figure 1. Arcsecond of Great Circle.

and using the polar radius, too small: it is the mean radius that is required. [8]

Mean 100' length from Parthenon	101.368 feet	
Mean Great Circle arcsec.	101.365 feet	
Error	0.003%	(1)

Thus, *one hundred Greek feet measures a Great Circle arcsecond*. Ten "Olympic" Greek feet times 60^4 equal a Great Circle's circumference. The Acropolis, glory of Athens and its highest point, gives us the most exact estimate of the length of a Greek foot. Six hundred of these comprised a stade, the length of a "stadium."[9] Ten stades equal a Great Circle's arcminute.

In more recent times, the "nautical mile" was defined as the length of a Great Circle arcminute. This length varied from pole to equator due to Earth's bulge at the equator, increasing with latitude (for reasons that may not be at once evident), and so its length was eventually defined as the length of an arcminute in the British Channel. If we plot the length in feet of an arcsecond of Great Circle with Earth's varying radius, from equator to pole, then around that latitude at 50° it becomes equal to the *hundred Greek feet* as defined by the Acropolis [10] (see figure).

Multiplying the Acropolis-width of a hundred Greek feet by sixty, therefore gives the English length of a nautical mile of 6080 feet, to within 99.96%.[11] The nautical mile is a *mean value* for the length of an arcminute, that can be drawn around the Great Circles of the Earth. We thereby gather that the Greek measure was in proportion to the overall, mean radius of the earth. This design of a temple to Minerva was an expression of the Greek concept of *Kosmos*, which meant a beauty as experienced in the totality of things: this word appeared in the 5th century BC, which was when the Acropolis was constructed.

Here the view of the late John Ivimy is advocated.[12] Greek mathematicians had no sexagesimal notation, unlike the Babylonian. That is to say there was no inherent reason why they would have wanted to make a stade equal six hundred feet. He suggested that they had inherited their system of units from an older culture, without realising that it was geodetically defined. Thus at the time of constructing the Acropolis in the 5th century BC, *somebody* must have known of this geodetic or earth-circumference based definition; whereas, by the time of Eratosthenes, endeavouring to measure the Earth's size at Alexandria, in the 3rd century BC, it had in large measure been forgotten.

Berriman's statement that "Certainly the Greeks were unaware that the circumference of the Earth measures 216,000 Greek stades"[13] would appear to be true of the entire corpus of Greek philosophy, i.e. they were in the condition of having forgotten the root source of their measurement system, as based upon Earth-measure.

This argument implies an ancient ability to measure distance somewhat as nautical miles in recent times, and it suggests some concept resembling that of longitude. It implies a division of the circle into 360 degrees, and then further into minutes and seconds, somewhat before this scheme is believed to have been invented. History tells us that the twelvefold division of the ecliptic into the zodiac happened in the 6th/5th centuries BC, and then its subdivision into 30 degree intervals in the 5th/4th centuries BC: it appeared in a stellar context, as the longitudes of stars were mapped out against the zodiac, in ancient Babylon. The complete circle of 360° appears in the 3rd/2nd centuries BC, and in Greece not Chaldea. Only later on was this division mapped out geographically. Further evidence for such a "too-early" subdivision of the circle, is hard to come by.

Base-sixty mathematics was, however, used in ancient Sumeria in the 3rd and 2nd millennia BC, as the earliest mathematics, and it used base-sixty divisions of time, eg. the course of the day, long before it was applied to space. To quote Ivimy:

> *"The Sumerian sexagesimal notation as it has come down to us in cuneiform texts was manifestly designed not, as one might expect an ancient number system to be designed, for the simple purpose of counting, but rather for calculating, and in particular for problems involving division."* [14]

It is a problem of division that we here contemplate, by the fourth power of sixty, to obtain the arcsecond measure.

The 24:25 Tie-up

There is agreement amongst classical authors, that the Greek and Roman feet were interlocked to a 24:25 ratio, with the Greek foot 4% longer than the Roman: this ratio was "universally accepted," according to Professor Bowsher. [15] Pliny thus ascribed 625 feet to a stade [16]

(The Roman and Greek stades had the same length, but contained different numbers of feet). In 1639, the Oxford astronomy professor John Greaves visited Rome, and took careful measures of the Roman foot from certain monuments in the Vatican garden. [17] He thereby estimated the Roman foot at 11.66 inches; and we may note that this, using the 24:25 ratio, gives a Greek foot in accord with what we have derived from the Acropolis' dimension, to one part in a thousand. Scholars have generally estimated the "Olympic" Greek foot at between 12.15 and 12.16 inches, i.e. they have differed by only one part in twelve hundred.[18]

Jomard's Hypothesis

Edmé-Francois Jomard was a mathematical and classical scholar. While journeying with Napoleon to Egypt in 1798, it dawned upon him that the different varieties of "stade" found in antiquity had come about by dividing a degree of Great Circle in different ways: *"Le stade n'est autre chose que le degree terrestre consideré comme unité et divisé de differentes manières."*[19]

For example, dividing by 600 gave the Greek and Egyptian stade, while dividing by 500 gave that which Claudius Ptolemy used in his *Geographica*. The attempt had been made to define the metre geodetically in the late 18th century (as 10,000th of a polar Great Circle quadrant), but the effort had been abortive because the Earth's size was not then known well enough. In 1812, in the aftermath of this failure, Jomard published his geodetic hypothesis.

Claudius Ptolemy in Alexandria composed his *Geographica*, which opus divided the Great Circles of Earth into 360°, and may have been the first to do so in a systematic manner.[20] There were, he affirmed, 500 stades per degree of latitude (i.e. Great Circle). Thus, concerning a location 63° due North he wrote: "Now the latitude he notes as measuring 31,500 stadia; since every degree, it is accepted, has 500 stadia." Later on he added, rather too briefly, that the equivalence of 500 stades per degree was "a measurement that is proved by distances that are known and certain."[21]

Eratosthenes, also in Alexandria and four centuries earlier, had concluded that 252,000 stades girdled the Earth, which means that he had 700 stades per degree of Great Circle. These two different stades look very much like angular measure, *as if* they had been defined by there being 500 or 700 of them per degree. Academic scholars of ancient geography cannot ever accept such an argument, because they cannot accept Jomard's hypothesis that the different stades were derived from subdivisions of "le degré terrestre," i.e., a 1° arc of Great Circle. Let us here refrain from entering into the morass of conflicting arguments, but merely observe, that in Alexandria, what look very much like geodetic definitions of the stade appeared; while in Athens, no mathematician was on record as averring that 600 of their stades measured out a degree of Great Circle — as was the case.

The Circle of Stonehenge

If perchance Stonehenge were to have a unit of length encoded within it, then it would have to be found in the inner diameter of its magnificent ring of thirty Sarsens. Why else would its lintels have been fitted so carefully, with cup-and-groove joins as if carved out of wood, and have their internal radius of a smaller curvature than their outer radius, and be positioned so that their top surface was more level than the ground on which the monument stood? Even though the lintel circle is now reduced to six stones out of the original thirty, its inner diameter remains identical to that of the Sarsen circle.

The monument was surveyed by the British archaeologist Flinders Petrie, who had a specially made lightweight surveying chain of his own design, that could be pulled taut across uneven ground for greater accuracy. The inner diameter of the Sarsen circle he found to average 1167.9 inches. This, he affirmed, was "recognised as 100 Roman feet."[22, 23]

Petrie's earlier opus *Inductive Metrology: the Recovery of Ancient Measures from the Monuments* (1877) had made no allusion to any geodetic basis for ancient units, and derived its induced values from diverse sources. His report on Stonehenge built upon these results, concluding that the foot used for the inner Sarsen circle diameter had been"closely accordant with the Roman foot, which, though 11.64 inches in Rome, had a mean value of 11.68 inches ±0.01 in Greece, Africa & England. Not that this shows Stonehenge to be post-Roman, as the unit was the great Etrurian and Cyclopaen unit, originally derived from Egypt, and it may have been introduced at any date into Britain."

Then, in 1956, the archaeologist Ronald Atkinson estimated the inner diameter of the Sarsen circle as averaging 97 $^1/_3$ feet. [24] Since this measurement, no further estimate has been published of the diameter of this precious, ruined circle, in the world's most visited monument. Let us therefore just take a mean value of these two measures:

Petrie 1877	97.308 ft.	
Atkinson 1956	97.333 ft.	
Mean	97.319 ft.	(2)

We estimate how long a hundred Roman feet would be, from the hundred Greek foot length derived from the Acropolis [1] by using the 24:25 ratio: which gives us a value of 97.313 ft. This ties up with our best estimate of the Sarsen inner ring diameter, [2] at around one part in 10,000. Furthermore, the Roman foot value here used is identical with that which Petrie concluded had been used through "Greece, Africa and England," viz 11.68 inches.

The ring of Sarsens is several centuries older than the giant trilithons which now stand inside it.[25] This finding from modern radiocarbon dating is in some respects paradoxical; and yet, it may enhance the likelihood that the former held a metrological function. The ring of Sarsens was constructed around forty-six centuries B.P., and one may not readily comprehend how such a "Roman foot" measurement could have such a great antiquity. We cannot evade this implication, though it lacks an easy resolution.

In the late 18th century, French endeavours to define their meter geodetically came unstuck over an error of 0.02% in Earth's size [26] — too great a discrepancy for their definition to endure. For comparison, Newton's value for Earth's mean radius had erred by 1%.[27] By the 19th century four-figure accuracy had been soundly attained in Earth's dimensions, and this, it has here been suggested, enabled ancient geodetic definitions of the foot and stade to be re-apprehended. Over the last decade or two, a five-figure accuracy of Earth's dimensions has been obtained, improving our ability to resolve these age-old issues, concerning the original basis of length measure.

Summary

- Greaves' measure of the Roman foot (1639) gave, assuming the 24:25 ratio, a Greek foot of 12.15 inches.
- Penrose's measure of the Acropolis length and breadth (1888) gave Greek foot measures of 12.160 inches and 12.167 inches.
- Taking the British Nautical mile of 6080 feet as an arcminute of Great Circle, would give one-hundredth of an arcsecond as 12.16 inches.
- The modern mean Earth-radius gives 12.164 inches from the geodetic definition.
- Thus, the Greek foot may be geodetically defined as 10 x 60^4 of a mean Great Circle circumference.

References

1. Algernon Berriman, *Historical Metrology*, 1956, 1969, p. xvi.
2. Edward Nicholson, *Men and Measures*, 1912.
3. J. Spedding, *The Works ... of Francis Bacon*, 1858-74, Vol 3 p140-1.
4. www.newtonproject.ic.ac.uk/texts/yah41_d.html.
5. F.C.Penrose, *An Investigation of the Principles of Athenian Architecture*, 1888 p.7.
6. Ibid, pp.12-13. Its breadth and length he found to be 101.336 and 228.145 feet.
7. Berriman (ref.1): ten stades form an arc minute of "any great circle regarded as a sphere," p.1.
8. Earth's mean radius is presently accepted as 6,372.78 km or 3959.87 miles, see eg http://en.wikipedia.org/wiki/Earth_radius; this is called, "quadratic mean radius."
9. The stadium in Athens, i.e. the distance between its starting and finishing lines, was 606 feet 10 inches: W.B.Dinsmoor, *The Architecture of Ancient Greece*, NY 1973, 251; Donald Engels, "The Length of Eratosthenes Stade," Amer. Jnl. Philology 1985, 106, 298-311, 308.
10. www.csgnetwork.com/degreelenllavcalc.html for distance of a degree in latitude vs. latitude.
11. The metrically-defined, international nautical mile is 6076 feet.
12. John Ivimy, "The Sphinx and the Megaliths," Abacus 1976, p.55.
13. Berriman (ref.1), p.1 [n.b., 216,000 = 360 x 60 x 10].
14. Ivimy (ref .2), p.54.
15. Harry F. Bowsher, "Ancient Metrology of the Aegean Archipelago: The Archipelago as a focus for interdisciplinary research," Unesco 1981, Ed. E Maula, 69-93,75.
16. *Pliny Natural History*, Book II Ch.XXI: Loeb Classical Library, p.229; Berriman (ref.1), p.117.
17. John Greaves, *A Discourse of the Roman Foot*, 1647, p.33; Works I, 1737: the Roman foot he found to be 1944 parts to 2000 of an English foot. Thus, "the best value" of the Roman foot found by Greaves was 11.664 inches: Nicholson (ref.2), p.18.
18. Nicholson (ref.2), p.4 cited 12.16 inches; "This length ... may be regarded as certain" (p.14); while Ivimy (ref.12) concluded that the Greek foot "has been established as equal to 12.15 English inches," p.62.
19. E.Jomard, *Mémoire sur le Système Métrique des Anciennes Égyptiens*, 1817, Paris p.104.
20. J.L. Berggren and A Jones, *Ptolemy's Geography*, 2000, p.14.
21. Claudius Ptolemy, *The Geography*, 1991, Dover NY Book I Chs 7,11, pp.29,33.
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23. A.Thom & A.S.Thom, *Megalithic Remains in Britain and Brittany*, 1978, made no allusion to this. Instead, Thom attempted to fit his "Megalithic yards" to the inner diameter of the Stonehenge sarsens, while admitting that this was not exact: A.Thom, Journal for History of Astronomy 1974, 5, 71-90; Thom & Thom, 1978, p.144.
24. R.Atkinson, *Stonehenge*, 1956, 1979, p.38.
25. R.Cleal, K.Walker & R.Montague, "Stonehenge and Its Landscape, 20th Century Excavations," English heritage 1995, p.534: Sarsen Circle 2850-2480 B.P., Sarsen Trilithons 2440-2100 B.P.
26. Earth's mean polar circumference was thus intended to be 40000 km, but is 40008.6 km.
27. Isaac Newton, *The Principia*, 3rd Edn 1726, Book III Prop 19: 3923 miles mean radius.

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